Electrical Circuit-I

6th Lecture

Current & Voltage Sources Conversions

By:

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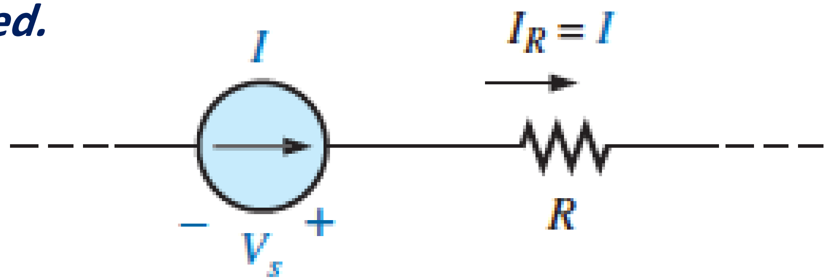
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Current Source

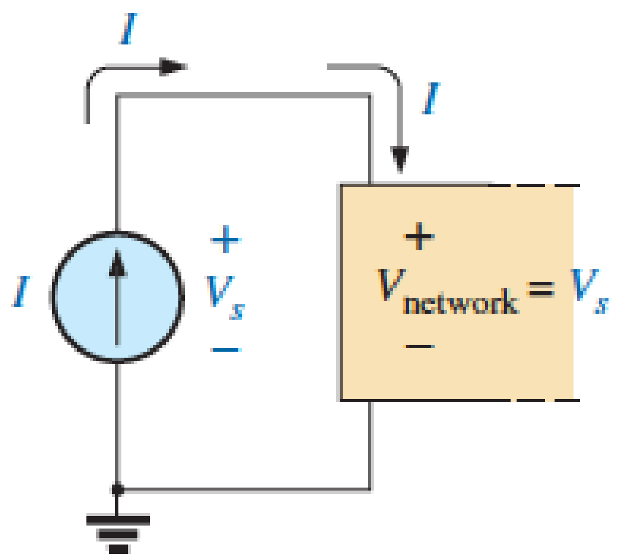
A current source determines the direction and magnitude of the current in the branch where it is located.

Furthermore, the magnitude and the polarity of the voltage across a current source are each a function of the network to which the voltage is applied.

The symbol for a current source appears in Fig. (a). The arrow indicates the direction in which it is supplying current to the branch where it is located. The result is a current equal to the source current through the series resistor. In Fig. (b), we find that the voltage across a current source is determined by the polarity of the voltage drop caused by the current source. For single-source networks, it always has the polarity of Fig. (b).



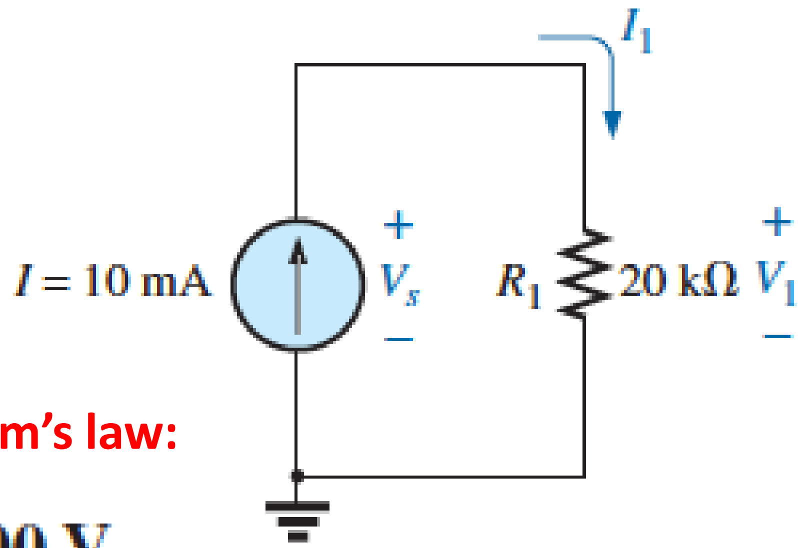
(a)



(b)

Example (1)

Find the source voltage, the voltage V_1, and current I_1 for the circuit



Solution:

$$I_1 = I = 10 \text{ mA}$$

The voltage across R1 is then determined by Ohm's law:

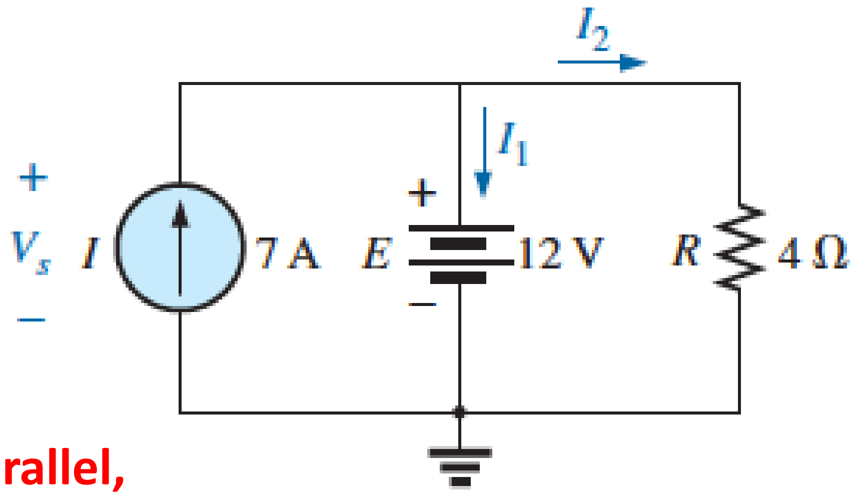
$$V_1 = I_1R_1 = (10 \text{ mA})(20 \ \Omega) = 200 \text{ V}$$

Since resistor R1 and the current source are in parallel, the voltage across each must be the same

$$V_s = V_1 = 200 \text{ V}$$

Example **(2)**

Find the source voltage V_S , and current I_1 & I_2 for the circuit



Solution:

Since the current source and voltage source are in parallel,

$$V_s = E = \mathbf{12\ V}$$

since the voltage source and resistor R are in parallel,

$$V_R = E = 12\ \text{V}$$

$$I_2 = \frac{V_R}{R} = \frac{12\ \text{V}}{4\ \Omega} = \mathbf{3\ A}$$

The current I_1 of the voltage source can then be determined by applying Kirchhoff's current law

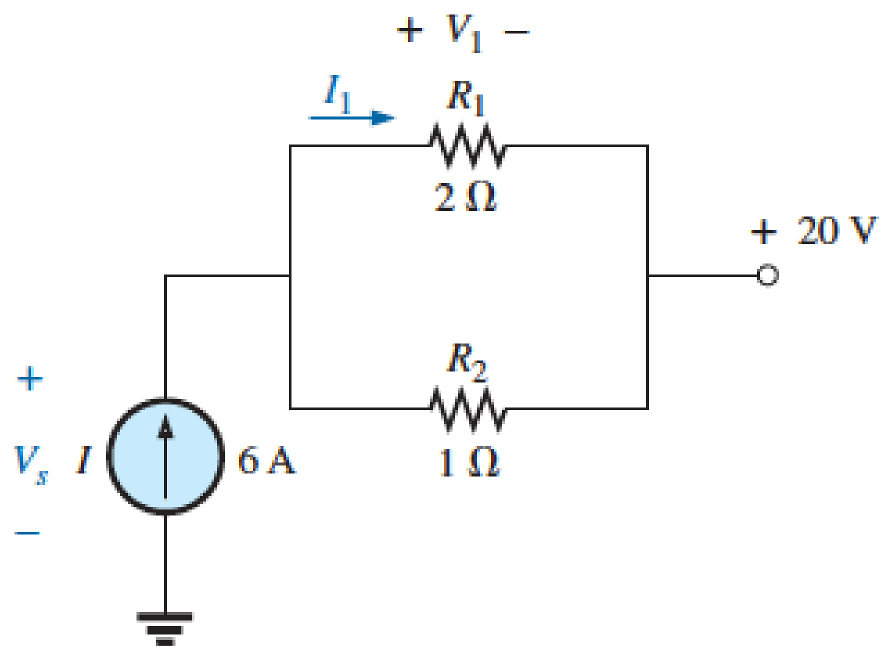
$$\Sigma I_i = \Sigma I_o$$

$$I = I_1 + I_2$$

$$I_1 = I - I_2 = 7\ \text{A} - 3\ \text{A} = \mathbf{4\ A}$$

***Example* (3)**

Determine the current I_1 and the voltage V_S for the network



Solution:

First note that the current in the branch with the current source must be 6 A, no matter what the magnitude of the voltage source to the right. In other words, the currents of the network are defined by I, R_1, and R_2. However, the voltage across the current source is directly affected by the magnitude and polarity of the applied source.

Using the current divider rule:
$$I_1 = \frac{R_2 I}{R_2 + R_1} = \frac{(1\ \Omega)(6\ \text{A})}{1\ \Omega + 2\ \Omega} = \frac{1}{3}(6\ \text{A}) = 2\ \text{A}$$

The voltage V_1 :
$$V_1 = I_1 R_1 = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

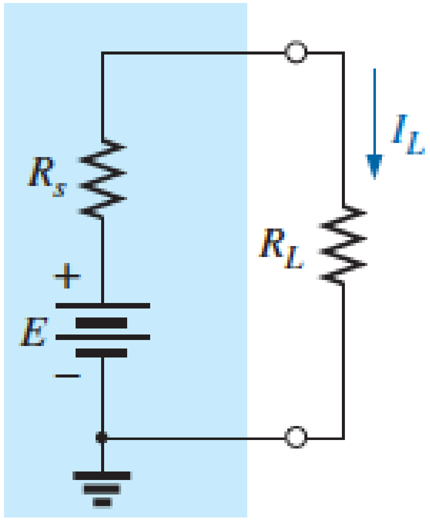
Applying Kirchhoff's voltage rule to determine V_S:

$$+V_s - V_1 - 20\ \text{V} = 0$$
$$V_s = V_1 + 20\ \text{V} = 4\ \text{V} + 20\ \text{V} = 24\ \text{V}$$

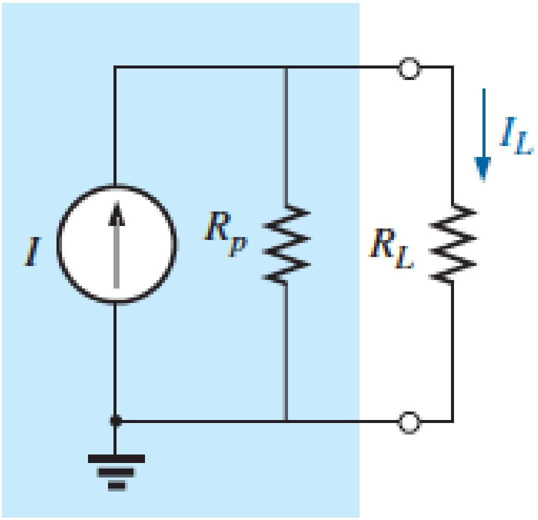
Source Conversions

The current source appearing is the previous section is called an ideal source due to the absence of any internal resistance. In reality, all sources—whether they are voltage sources or current sources—have some internal resistance in the relative positions shown below. For the voltage source, if $R_S = 0$, or if it is so small compared to any series resistors that it can be ignored, then we have an "ideal" voltage source for all practical purposes. For the current source, since the resistor R_P is in parallel, if $R_P = \infty$, or if it is large enough compared to any parallel resistive elements that it can be ignored, then we have an "ideal" current source.

Unfortunately, however, ideal sources cannot be converted from one type to another. That is, a voltage source cannot be converted to a current source, and vice versa—the internal resistance must be present.



(a)



(b)

Source Conversions

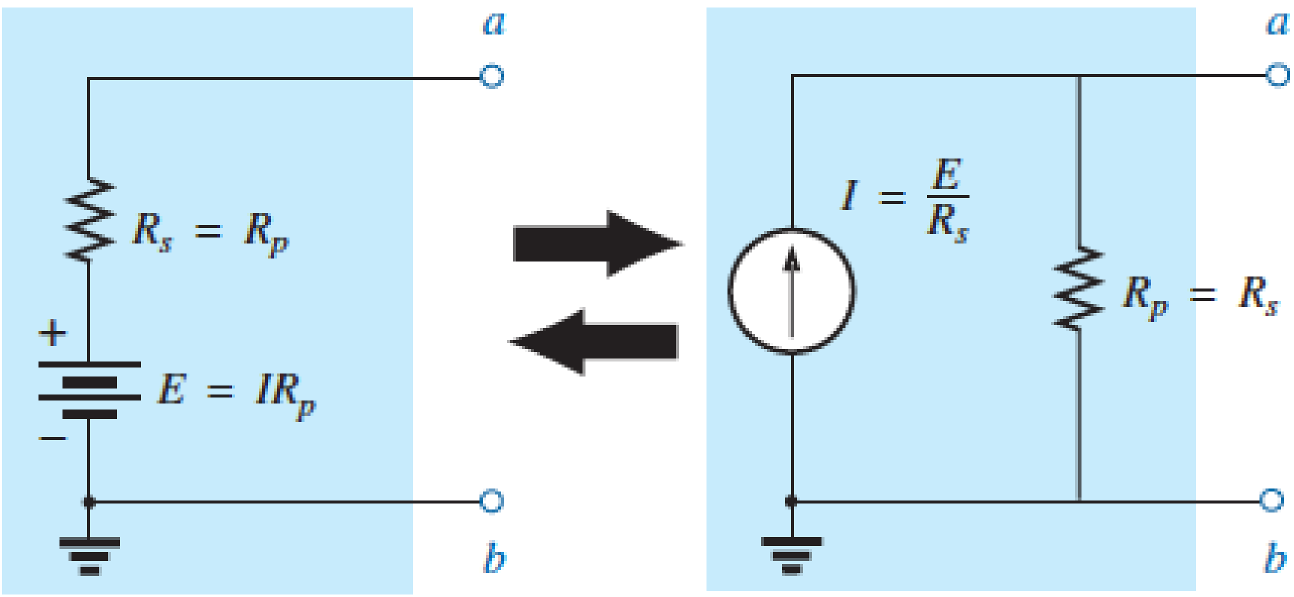
For the voltage source equivalent, the voltage is determined by a simple application of Ohm's law to the current source:

$$E = IR_p$$

For the current source equivalent, the current is again determined by applying Ohm's law to the voltage source:

$$I = E/R_s$$

The equivalence between a current source and a voltage source exists only at their external terminals.



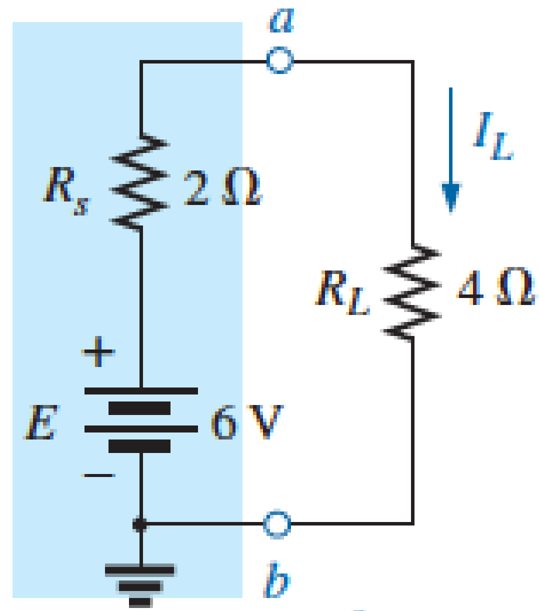
***Example* (4)**

For the circuit:

a. Determine the current I_L.

b. Convert the voltage source to a current source.

c. Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).



Solution:

a. Applying Ohm's law:

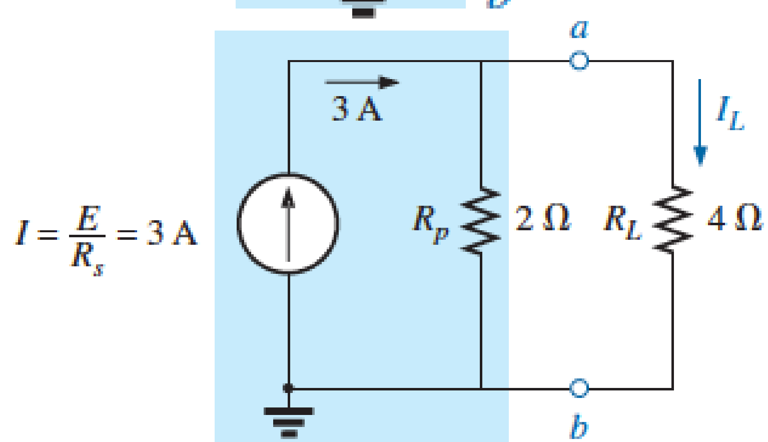
$$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = 1\text{ A}$$

b. Using Ohm's law again:

$$I = \frac{E}{R_s} = \frac{6\text{ V}}{2\ \Omega} = 3\text{ A}$$

c. Using the current divider rule:

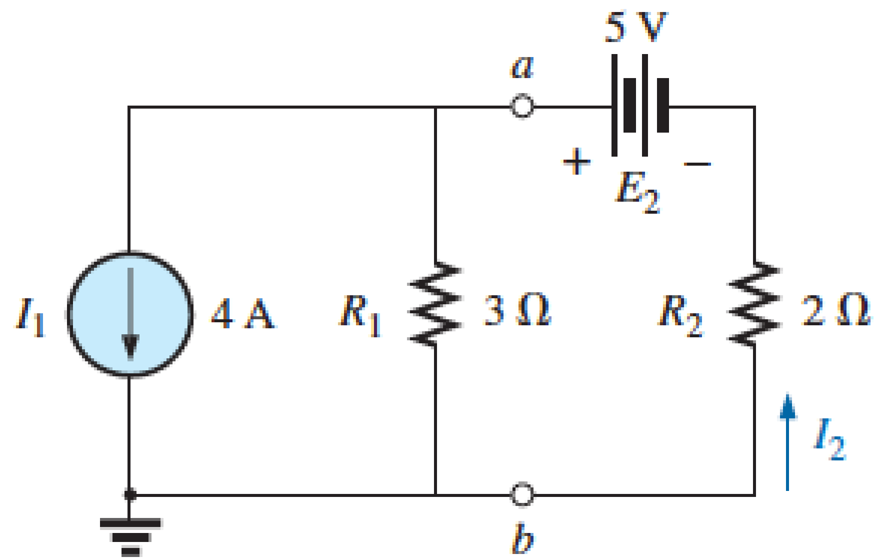
$$I_L = \frac{R_p I}{R_p + R_L} = \frac{(2\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \frac{1}{3}(3\text{ A}) = 1\text{ A}$$



We find that the current I_L is the same for the voltage source as it was for the equivalent current source the sources are therefore equivalent.

***Example* (5)**

Determine current I_2 for the network:



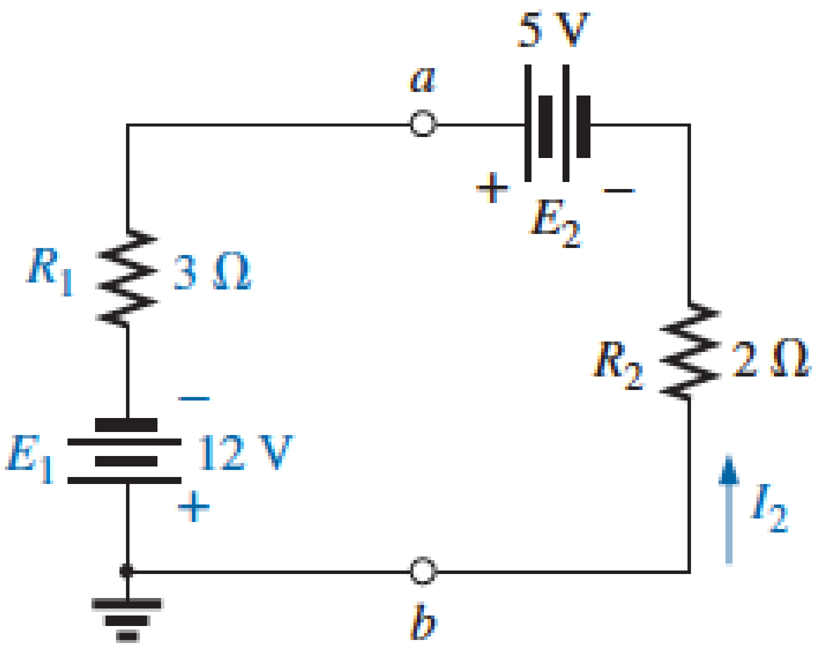
Solution:

Note the polarity for the equivalent voltage source as determined by the current source.

For the source conversion:

$$E_1 = I_1R_1 = (4\ \text{A})(3\ \Omega) = 12\ \text{V}$$

$$I_2 = \frac{E_1 + E_2}{R_1 + R_2} = \frac{12\ \text{V} + 5\ \text{V}}{3\ \Omega + 2\ \Omega} = \frac{17\ \text{V}}{5\ \Omega} = 3.4\ \text{A}$$



Current Source in Parallel

We found that "voltage sources of different terminal voltages cannot be placed in parallel because of a violation of Kirchhoff's voltage law".

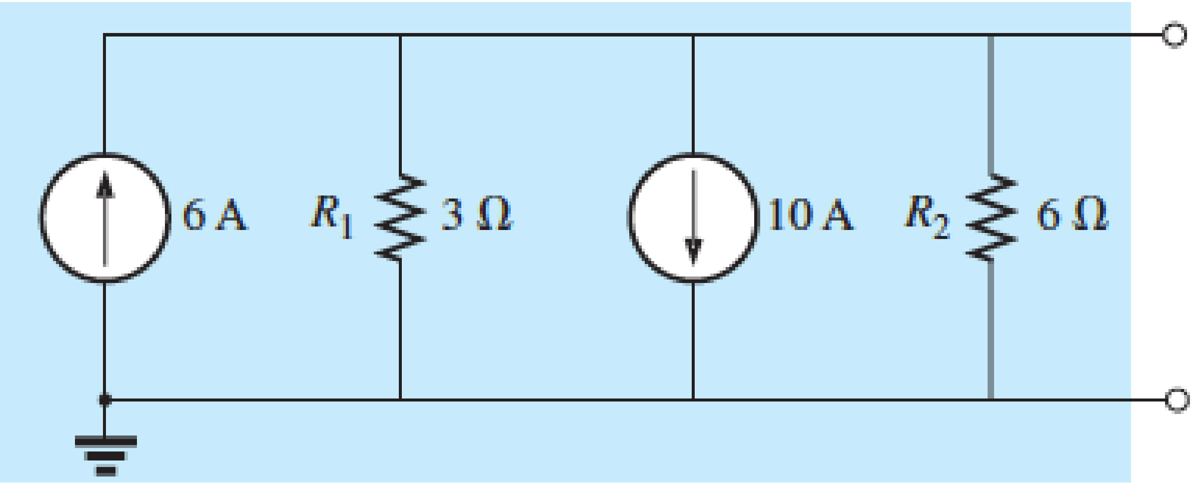
Similarly, "current sources of different values cannot be placed in series due to a violation of Kirchhoff's current law".

However, current sources can be placed in parallel just as voltage sources can be placed in series. In general,

Two or more current sources in parallel can be replaced by a single current source having a magnitude determined by the difference of the sum of the currents in one direction and the sum in the opposite direction. The new parallel internal resistance is the total resistance of the resulting parallel resistive elements.

Example (6)

Reduce the parallel current sources in the network to a single current source



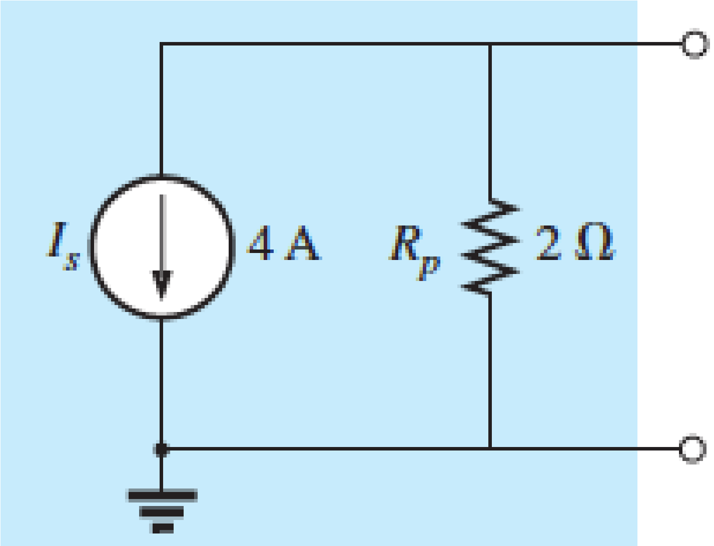
Solution:

The net source current is:

$$I = 10\text{ A} - 6\text{ A} = 4\text{ A}$$

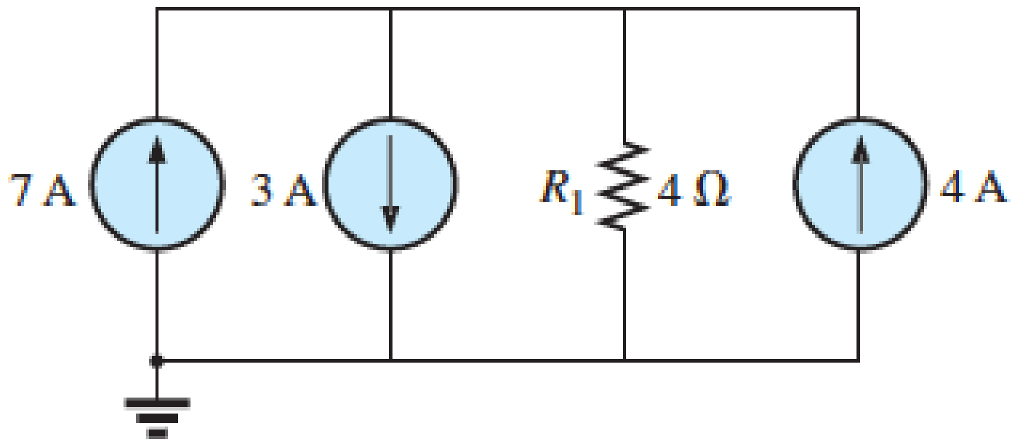
The net internal resistance is the parallel combination of resistors, R_1 *and* R_2 :

$$R_p = 3\,\Omega \parallel 6\,\Omega = 2\,\Omega$$



Example (7)

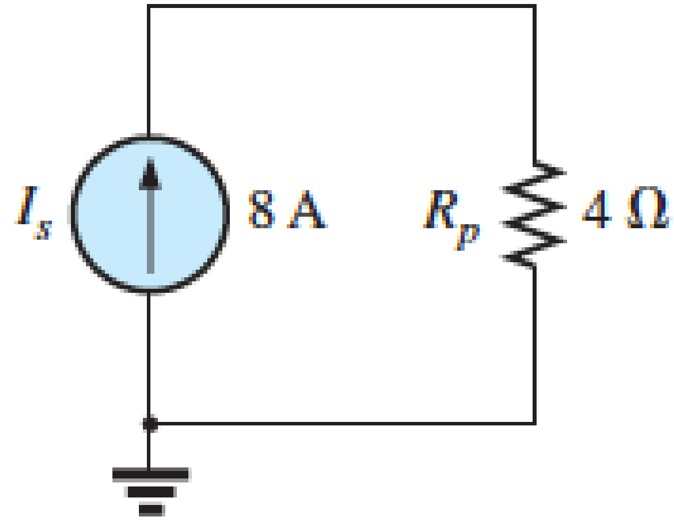
Reduce the parallel current sources in the network to a single current source



Solution:

The net source current is:

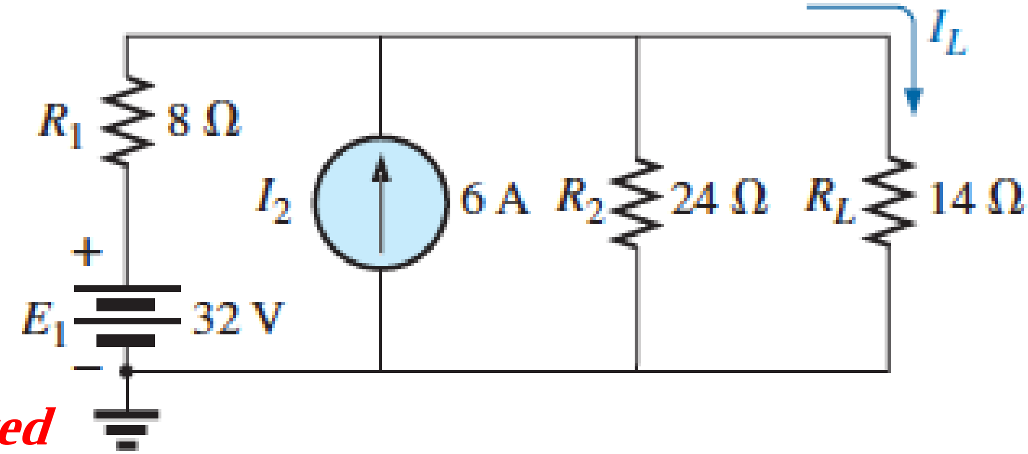
$$I = 7\text{ A} + 4\text{ A} - 3\text{ A} = 8\text{ A}$$



The net internal resistance remains the same.

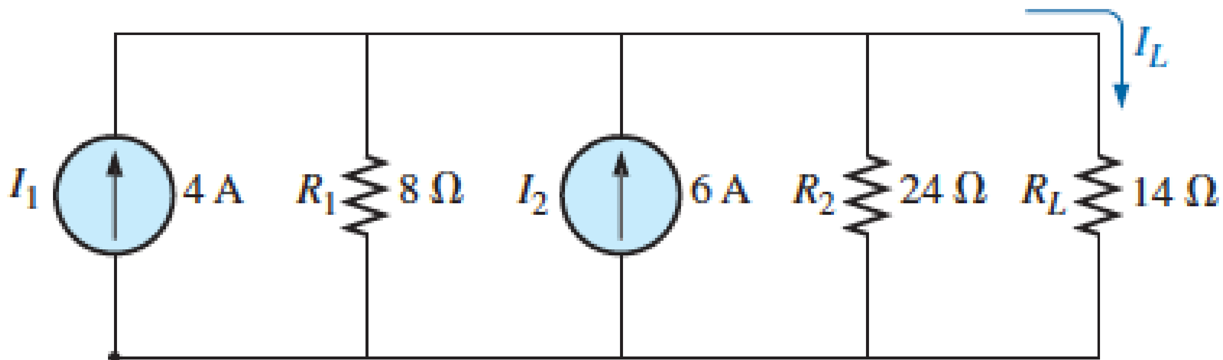
***Example* (8)**

Reduce the network to a single current source, and calculate the current through R_L



Solution:

the voltage source will first be converted to a current source as shown

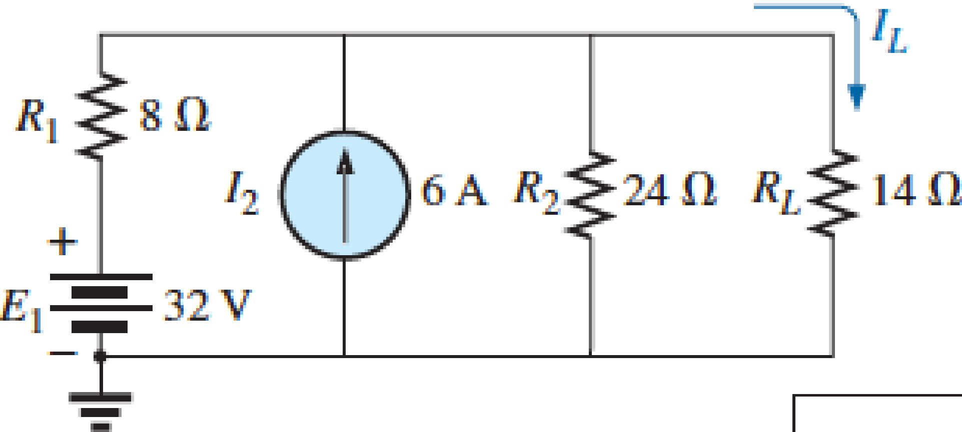


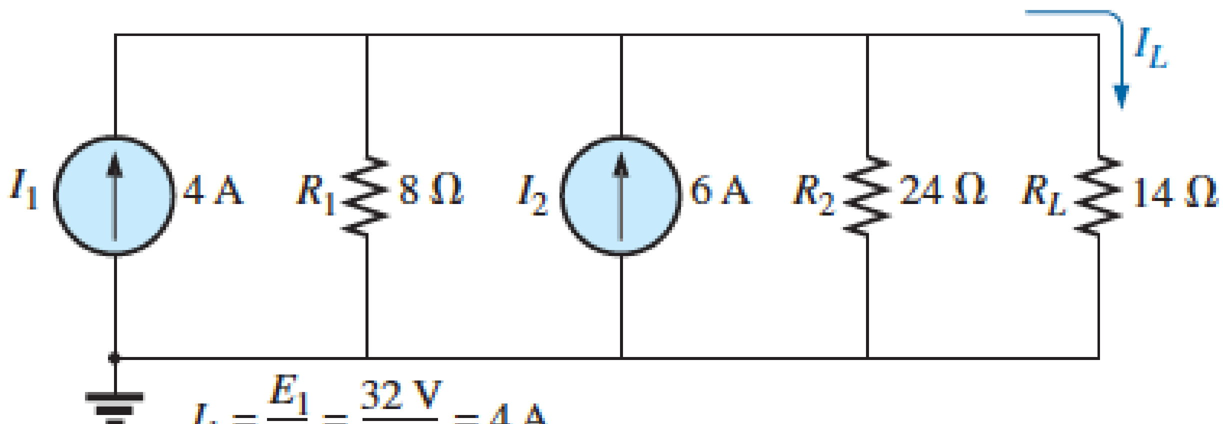
$$I_1 = \frac{E_1}{R_1} = \frac{32\ \text{V}}{8\ \Omega} = 4\ \text{A}$$

$$I_s = I_1 + I_2 = 4\ \text{A} + 6\ \text{A} = 10\ \text{A}$$

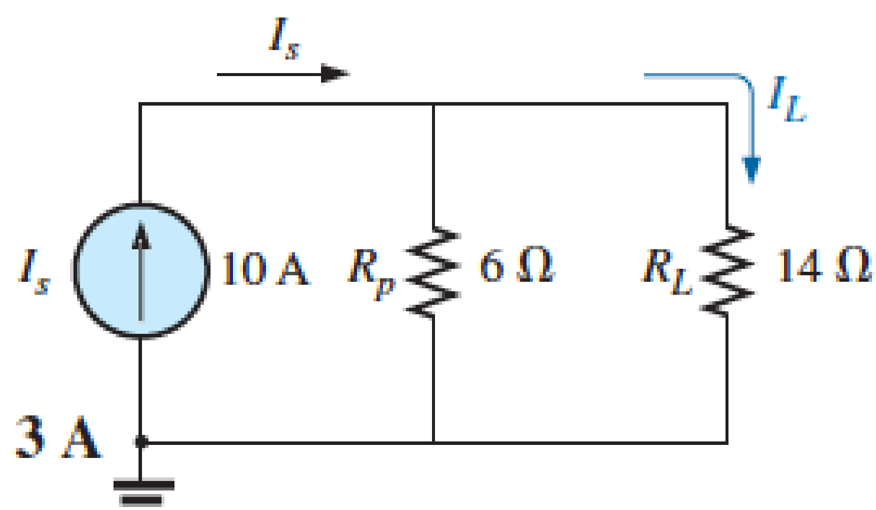
$$R_s = R_1 \| R_2 = 8\ \Omega \| 24\ \Omega = 6\ \Omega$$

Example (7)





$$I_1 = \frac{E_1}{R_1} = \frac{32\ \text{V}}{8\ \Omega} = 4\ \text{A}$$



Applying the current divider rule to the resulting network

$$I_L = \frac{R_p I_s}{R_p + R_L} = \frac{(6\ \Omega)(10\ \text{A})}{6\ \Omega + 14\ \Omega} = \frac{60\ \text{A}}{20} = 3\ \text{A}$$

Current Source in Series

The current through any branch of a network can be only single-valued. For the situation indicated at point a in Figure below, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering—an impossible situation. Therefore,

- ***Current sources of different current ratings are not connected in series,***
- ***just as voltage sources of different voltage ratings are not connected in parallel.***