Electrical Circuit-I

7th Lecture

Superposition Theorem

By:

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Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Superposition Theorem

The superposition theorem is unquestionably one of the most powerful in this field. It has such widespread application that people often apply it without recognizing that their maneuvers are valid only because of this theorem.

In general, the theorem can be used to do the following:

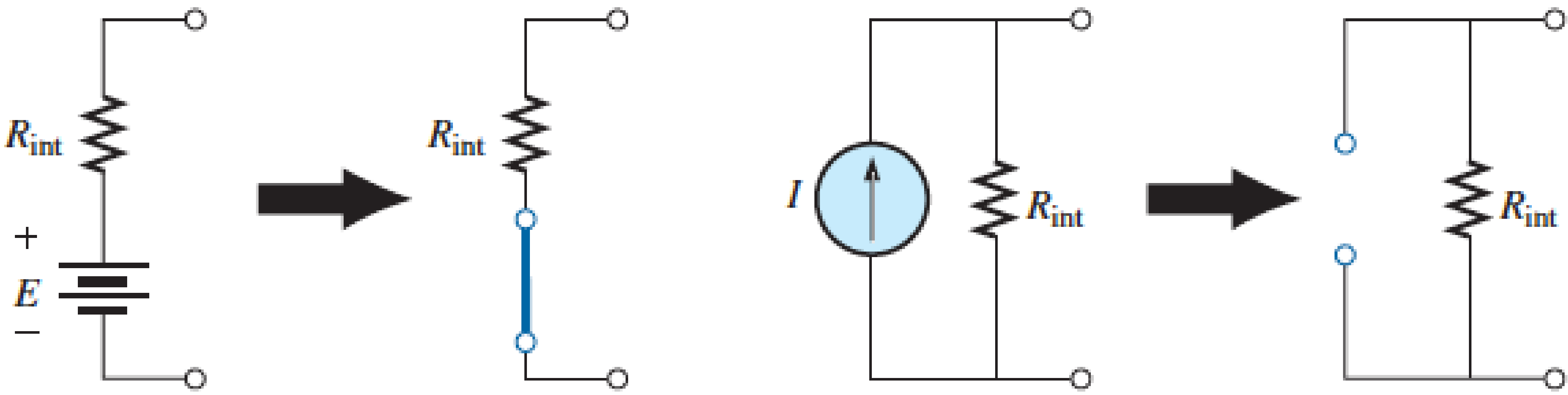
- *The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.*

In other words, this theorem allows us to find a solution for a current or voltage using only one source at a time.

- *when removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network.*
- *when removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.*

Superposition Theorem

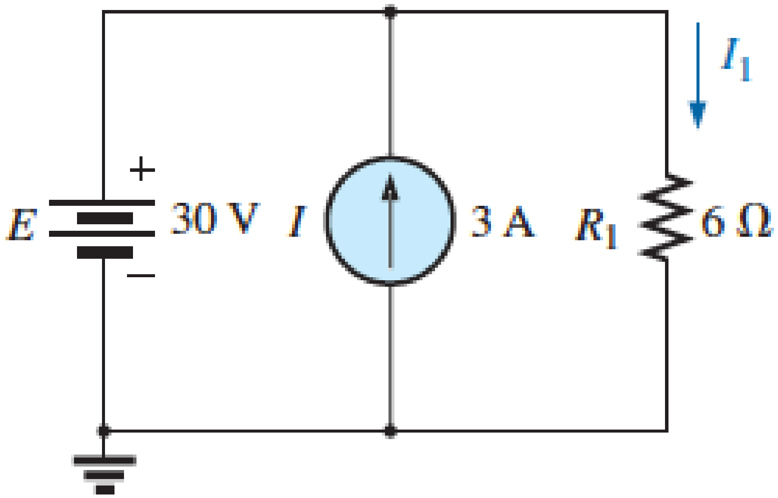
The above statements are illustrated below:



Since the effect of each source will be determined independently, the number of networks to be analysed will equal the number of sources.

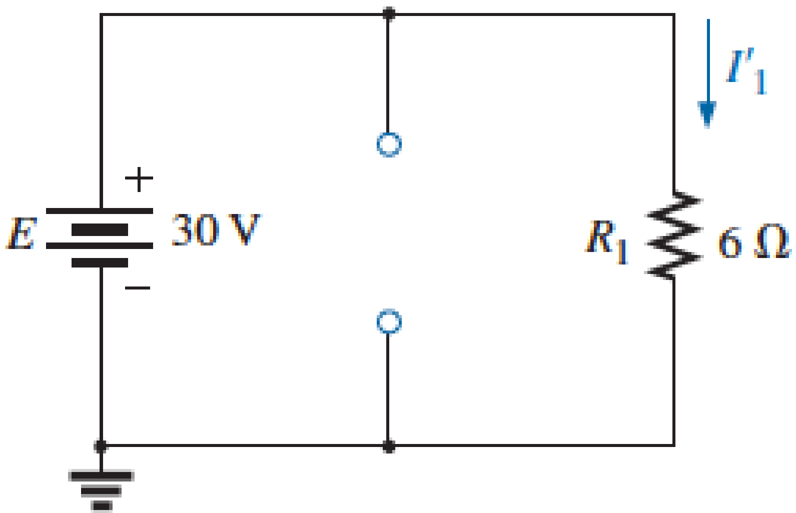
***Example* (1)**

Using the superposition theorem, determine current I_1 for the network



Solution:

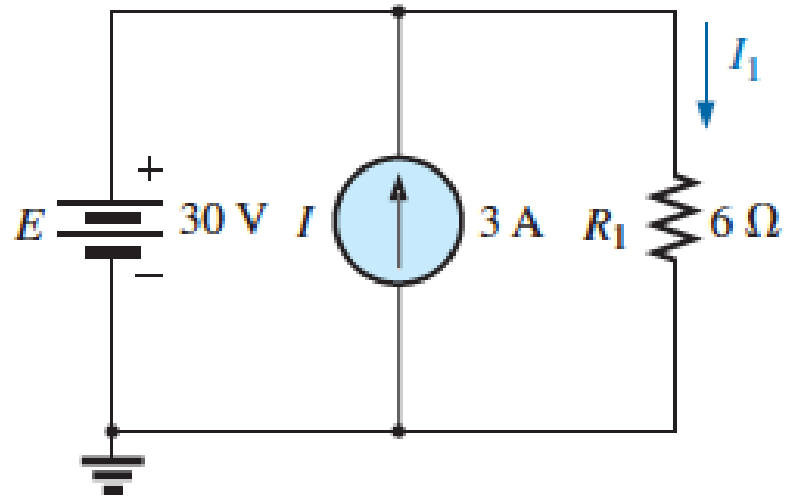
First let us determine the effects of the voltage source by setting the current source to zero amperes as shown



$$I'_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\ \text{V}}{6\ \Omega} = 5\ \text{A}$$

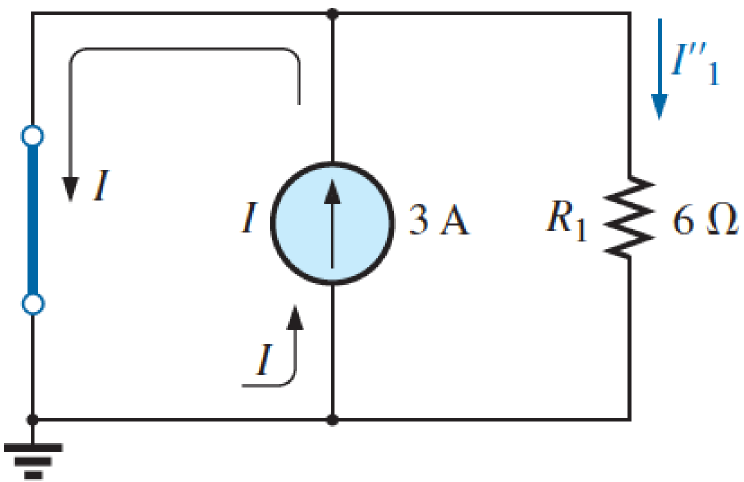
Now for the contribution due to the current source. Setting the voltage source to zero volts results as shown

Example (1)



Solution:

Now for the contribution due to the current source. Setting the voltage source to zero volts results as shown



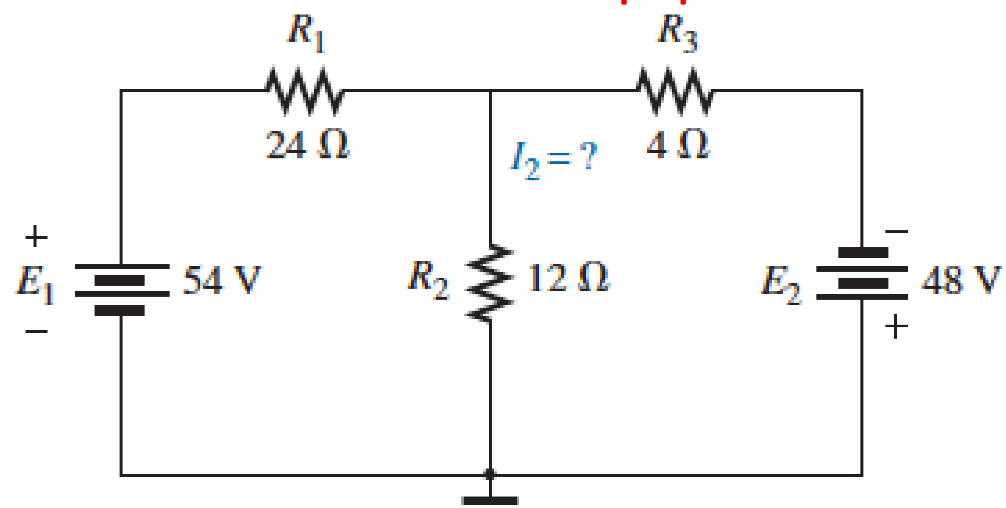
$$I''_1 = \frac{R_{sc}I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\ \text{A}$$

Since I'_1 and I''_1 have the same defined direction the total current is defined by

$$I_1 = I'_1 + I''_1 = 5\ \text{A} + 0\ \text{A} = 5\ \text{A}$$

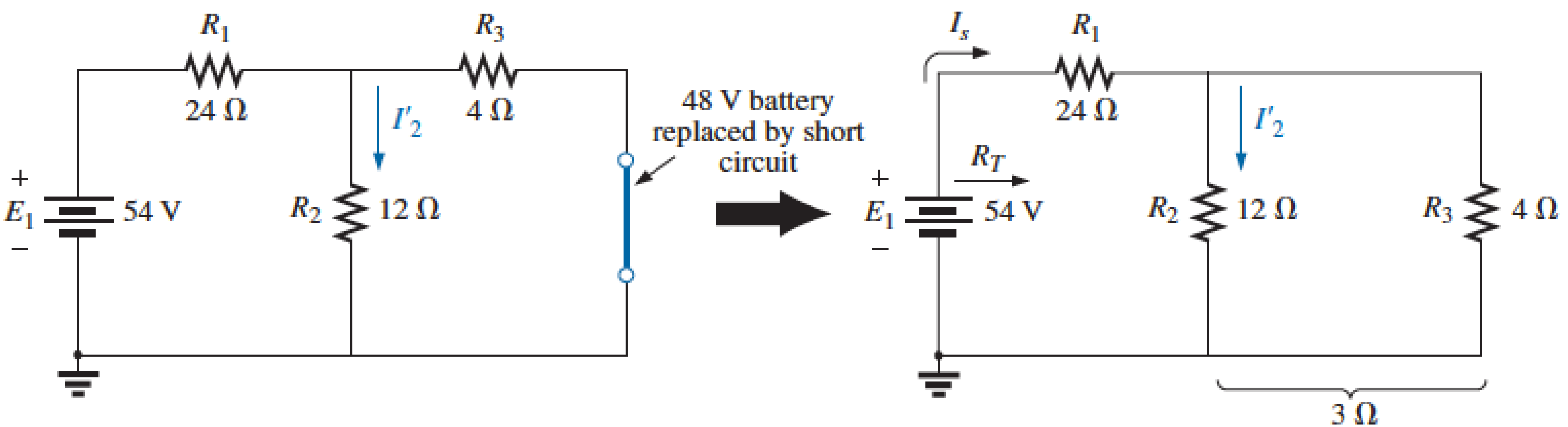
***Example* (2)**

Using the superposition theorem, determine the current through the 12 Ω resistor



Solution:

1) $E_2 = 0$



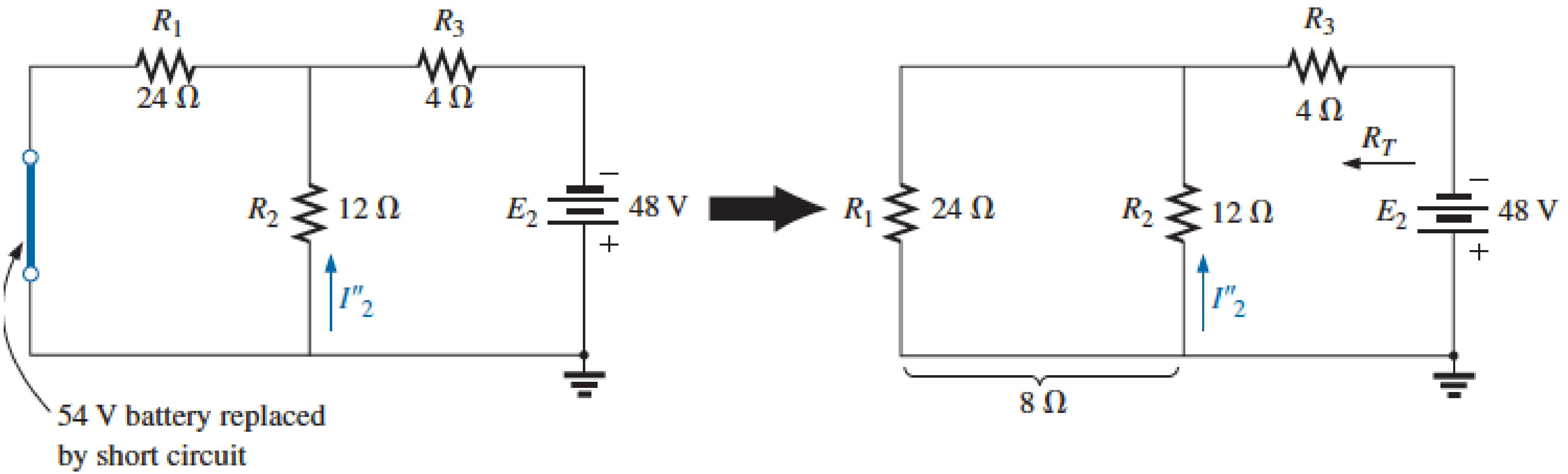
The total resistance seen by the source is therefore

$$R_T = R_1 + R_2 \| R_3 = 24\ \Omega + 12\ \Omega \| 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

$$I_s = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A} \quad \Rightarrow \quad I'_2 = \frac{R_3 I_s}{R_3 + R_2} = \frac{(4\ \Omega)(2\ \text{A})}{4\ \Omega + 12\ \Omega} = 0.5\ \text{A}$$

Example (2)

$2)\ E_1 = 0$

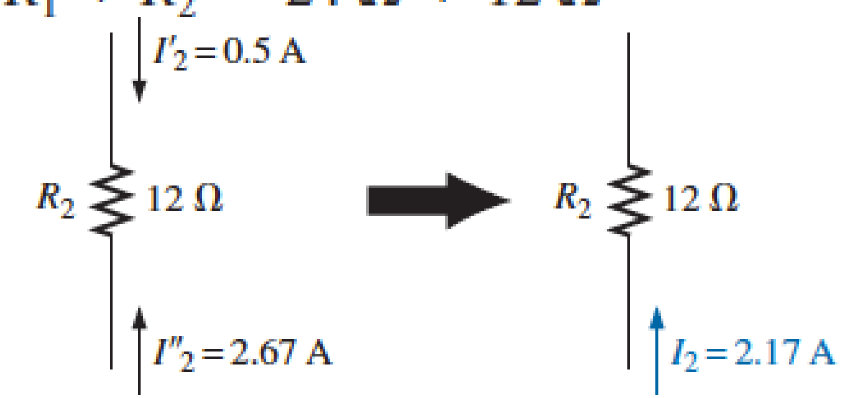


The total resistance seen by the source is therefore

$$R_T = R_3 + R_2 \| R_1 = 4\,\Omega + 12\,\Omega \| 24\,\Omega = 4\,\Omega + 8\,\Omega = 12\,\Omega$$

$$I_s = \frac{E_2}{R_T} = \frac{48\text{ V}}{12\,\Omega} = 4\text{ A}$$

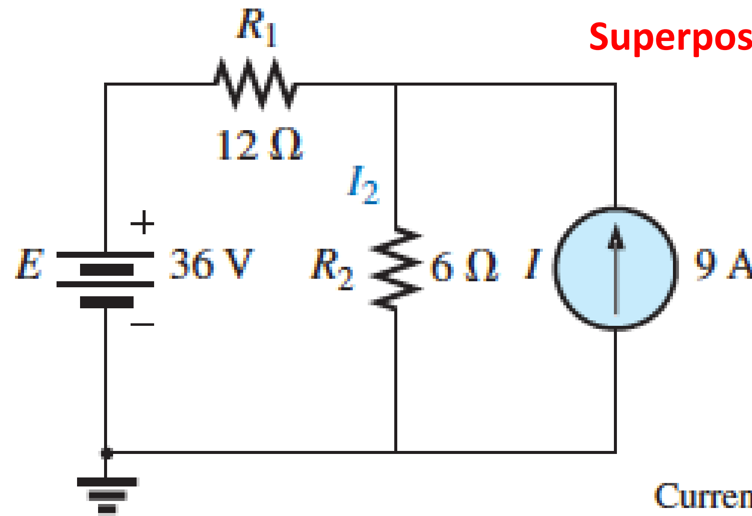
$$I''_2 = \frac{R_1(I_s)}{R_1 + R_2} = \frac{(24\,\Omega)(4\text{ A})}{24\,\Omega + 12\,\Omega} = 2.67\text{ A}$$



$$I_2 = I''_2 - I'_2 = 2.67\text{ A} - 0.5\text{ A} = \mathbf{2.17\text{ A}}$$

Example (3)

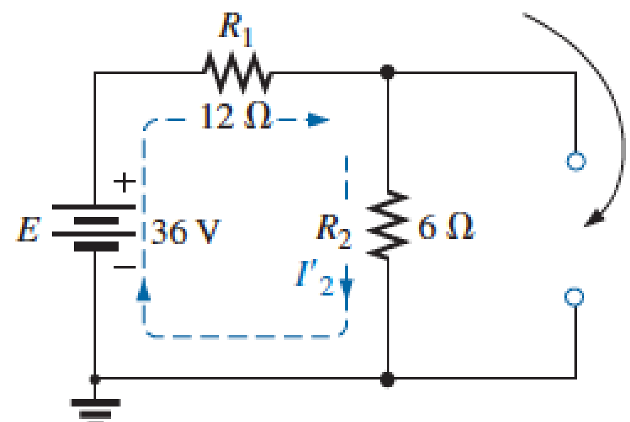
Using the superposition theorem, determine the current through resistor R_2 *for the network*



Solution:

1) $I = 0$

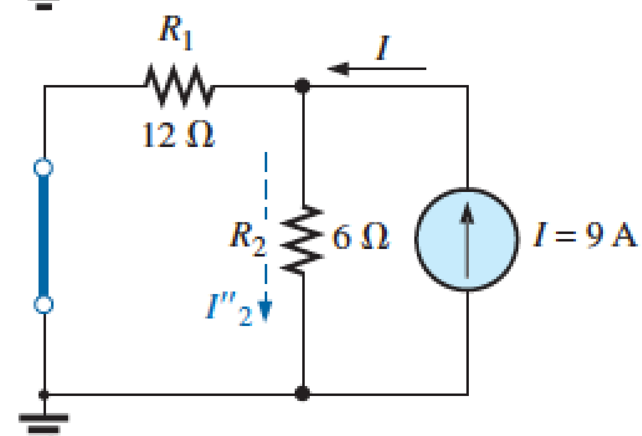
Current source replaced by open circuit



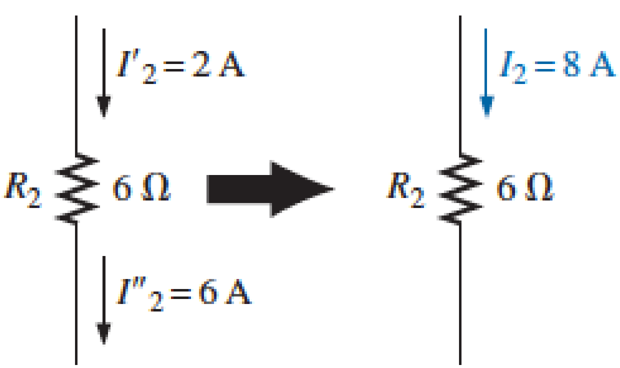
$$I'_2 = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\text{ V}}{12\ \Omega + 6\ \Omega} = \frac{36\text{ V}}{18\ \Omega} = 2\text{ A}$$

2) $E = 0$

$$I''_2 = \frac{R_1(I)}{R_1 + R_2} = \frac{(12\ \Omega)(9\text{ A})}{12\ \Omega + 6\ \Omega} = 6\text{ A}$$

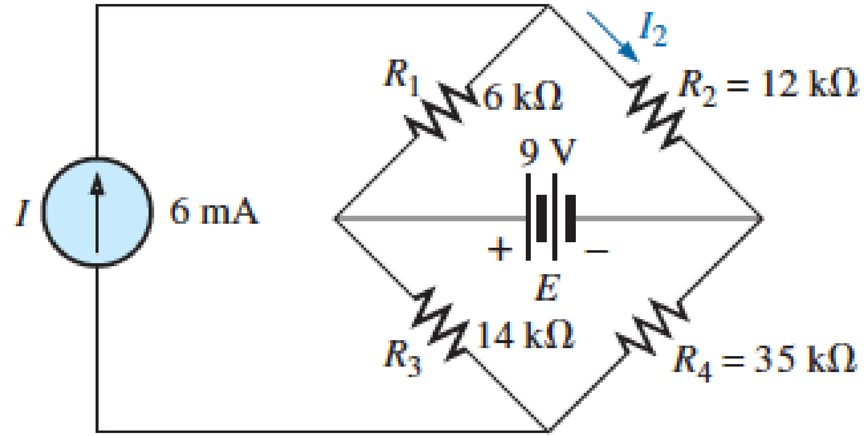


$$I_2 = I'_2 + I''_2 = 2\text{ A} + 6\text{ A} = 8\text{ A}$$



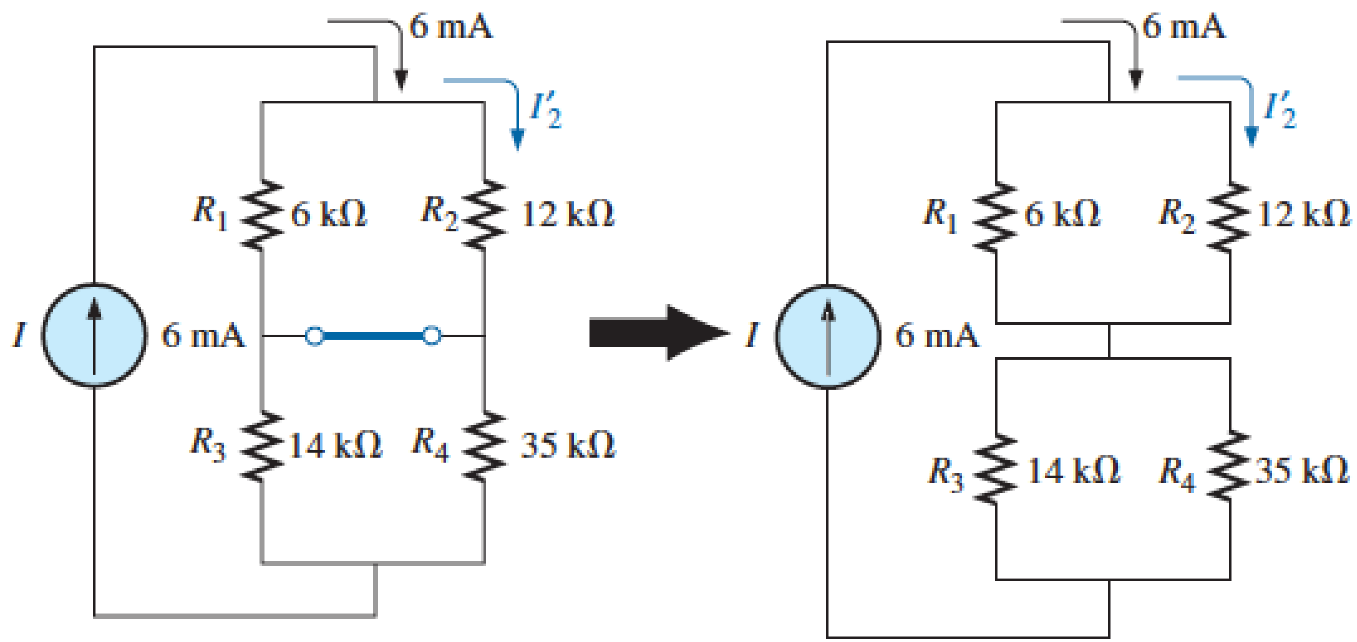
Example (4)

Using the principle of superposition, find the current I_2 *through the* **12 KΩ** *resistor*



Solution:

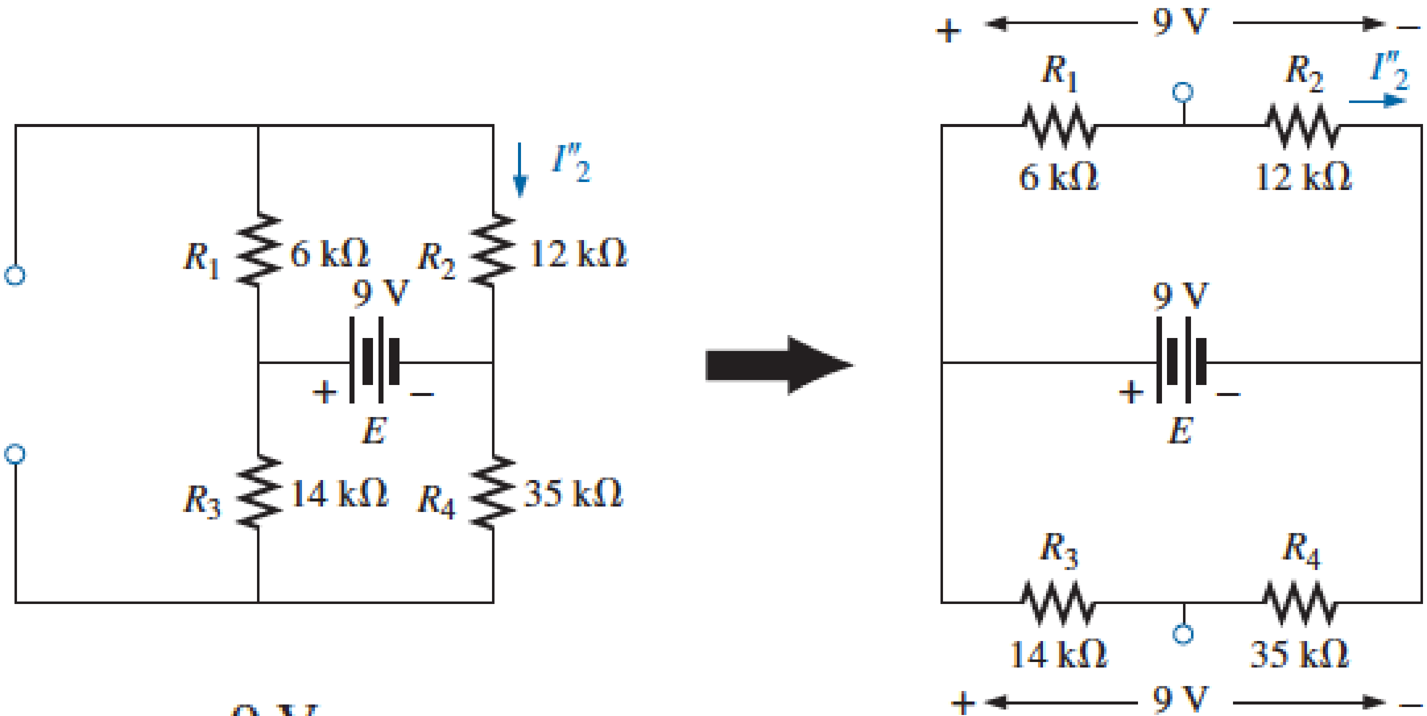
1) $E = 0$



$$I'_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(6\text{ k}\Omega)(6\text{ mA})}{6\text{ k}\Omega + 12\text{ k}\Omega} = 2\text{ mA}$$

Example (4)

2) $I = 0$



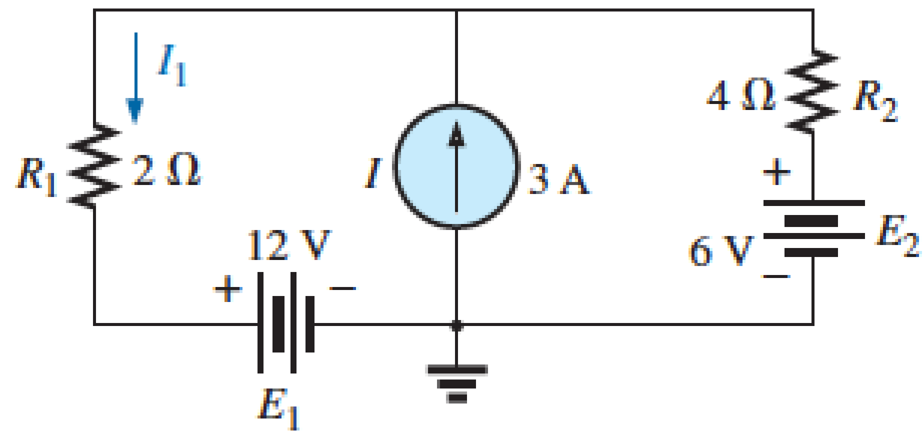
$$I''_2 = \frac{E}{R_1 + R_2} = \frac{9\text{ V}}{6\text{ k}\Omega + 12\text{ k}\Omega} = 0.5\text{ mA}$$

Since I'_2 and I''_2 have the same direction through R_2, the desired current is the sum of the two:

$$\begin{aligned} I_2 &= I'_2 + I''_2 \\ &= 2\text{ mA} + 0.5\text{ mA} \\ &= 2.5\text{ mA} \end{aligned}$$

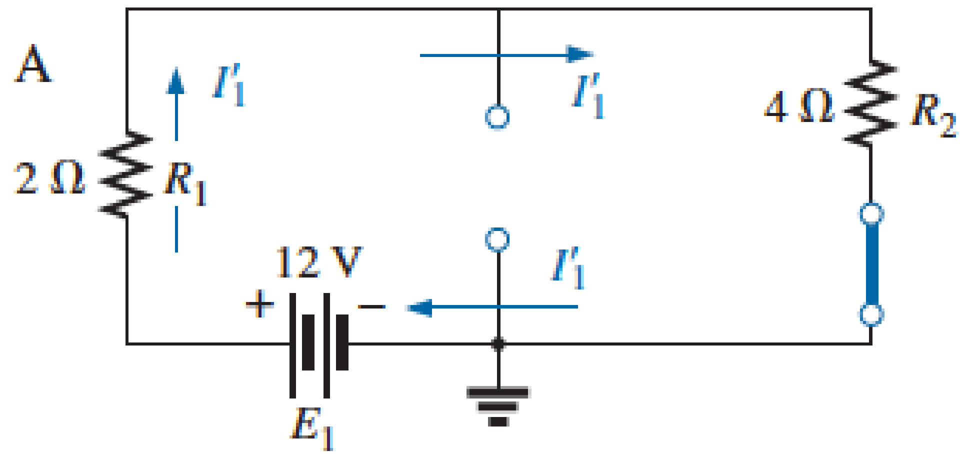
Example (5)

Find the current through the **2 Ω** *resistor of the network*



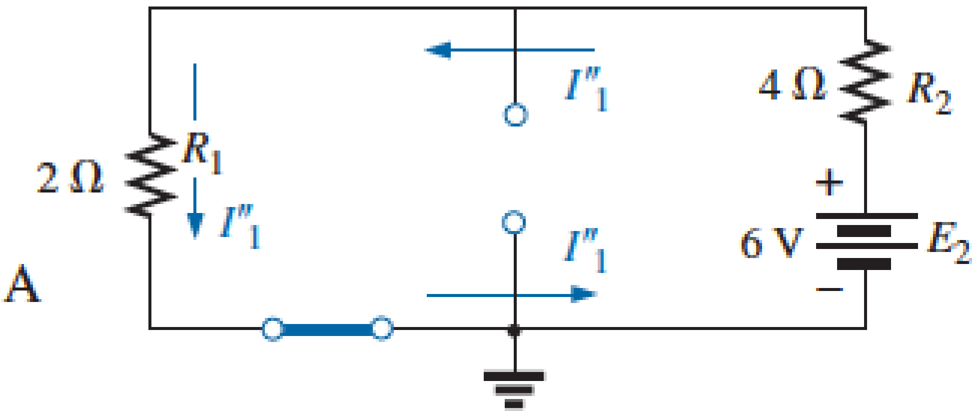
Solution:

1) $I = 0$ *and* $E_2 = 0$



$$I'_1 = \frac{E_1}{R_1 + R_2} = \frac{12\ \text{V}}{2\ \Omega + 4\ \Omega} = \frac{12\ \text{V}}{6\ \Omega} = 2\ \text{A}$$

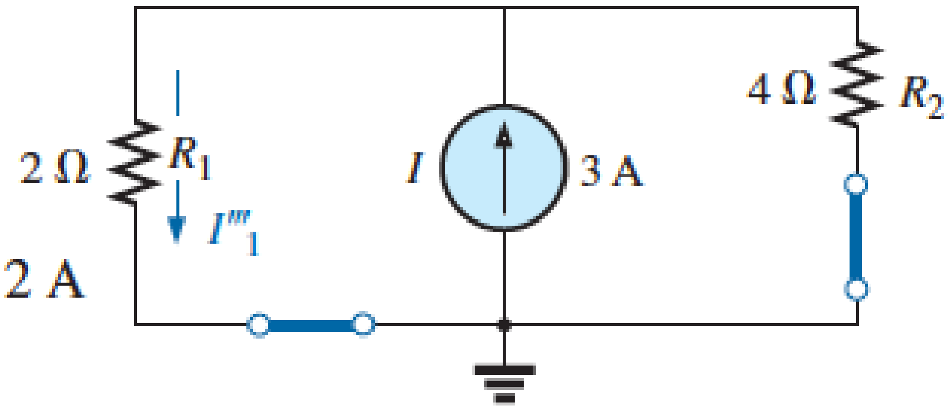
2) $I = 0$ *and* $E_1 = 0$



$$I''_1 = \frac{E_2}{R_1 + R_2} = \frac{6\ \text{V}}{2\ \Omega + 4\ \Omega} = \frac{6\ \text{V}}{6\ \Omega} = 1\ \text{A}$$

Example (5)

$3)\ E_1 = 0 \ \ and \ \ E_2 = 0$



$$I'''_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\ \Omega)(3\ \mathrm{A})}{2\ \Omega + 4\ \Omega} = \frac{12\ \mathrm{A}}{6} = 2\ \mathrm{A}$$

The total current through the* 2 Ω *resistor

Opposite direction to I_1 in Fig. 9.18

$$I_1 = \overbrace{I''_1 + I'''_1}^{\text{Same direction as } I_1 \text{ in Fig. 9.18}} - I'_1$$

$$= 1\ \mathrm{A} + 2\ \mathrm{A} - 2\ \mathrm{A} = 1\ \mathrm{A}$$