Electrical Circuit-I

8th Lecture

Thévenin's Theorem

By:

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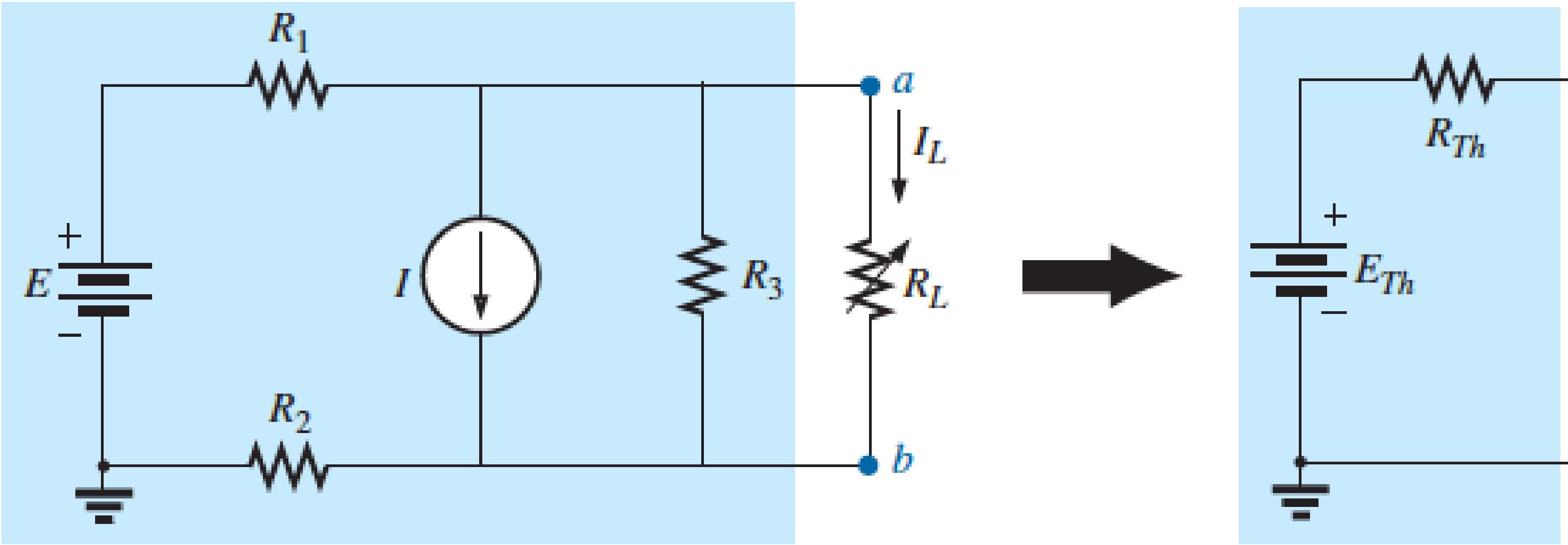
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Thévenin's Theorem

The theorem states the following:

Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor as shownin below.



Thévenin 's Theorem Procedure

1 Remove that portion of the network across which the Thévenin equivalent circuit is found.

2 Mark the terminals of the remaining two-terminal network.

R_{TH}

3 Calculate R_{TH} by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

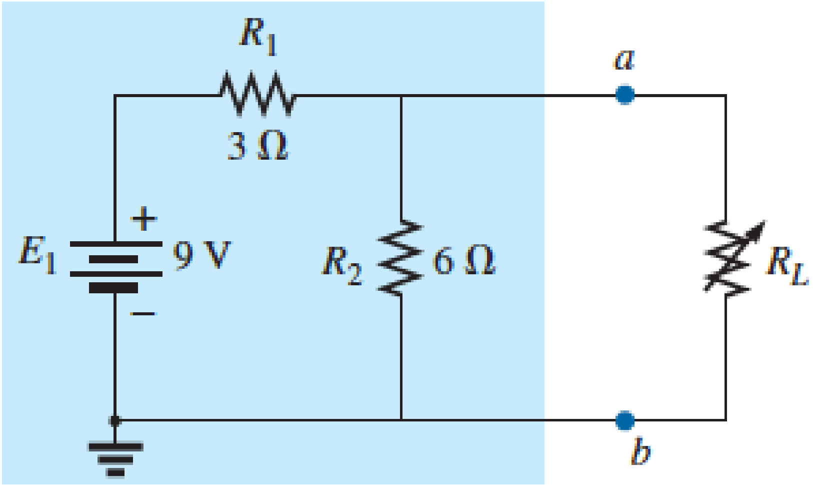
E_{TH}

4 Calculate E_{TH} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the open circuit potential between the two terminals marked in step 2.)

5 Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

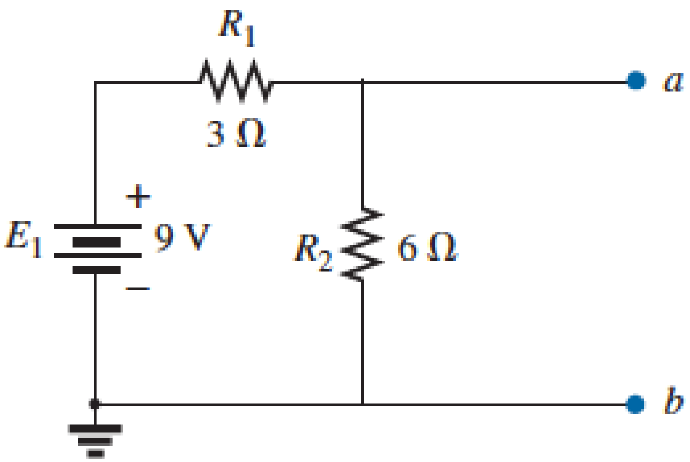
Example (1)

Find the Thévenin equivalent circuit for the network in the shaded area of the network . Then find the current through R_L for values of 2Ω, 10 Ω, and 100 Ω.

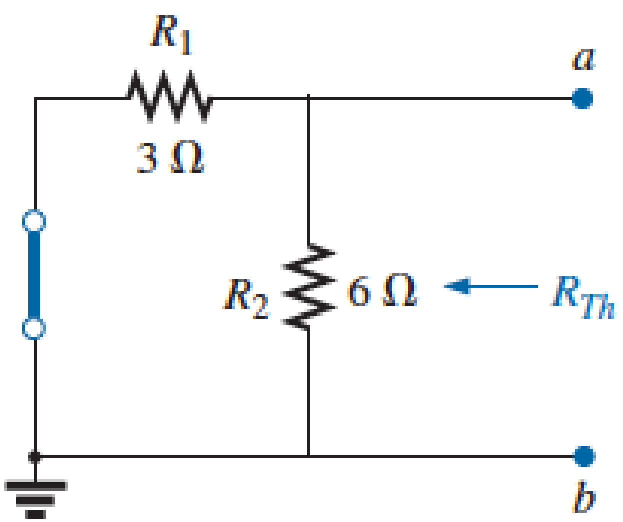


Solution:

Steps 1 and 2:



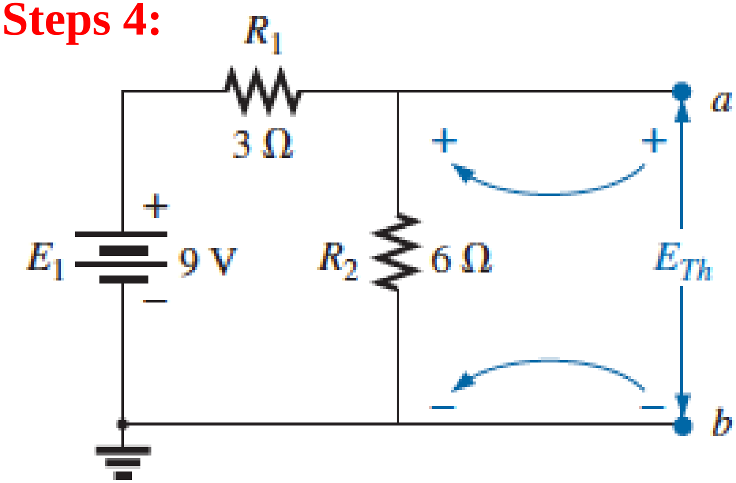
Steps 3:

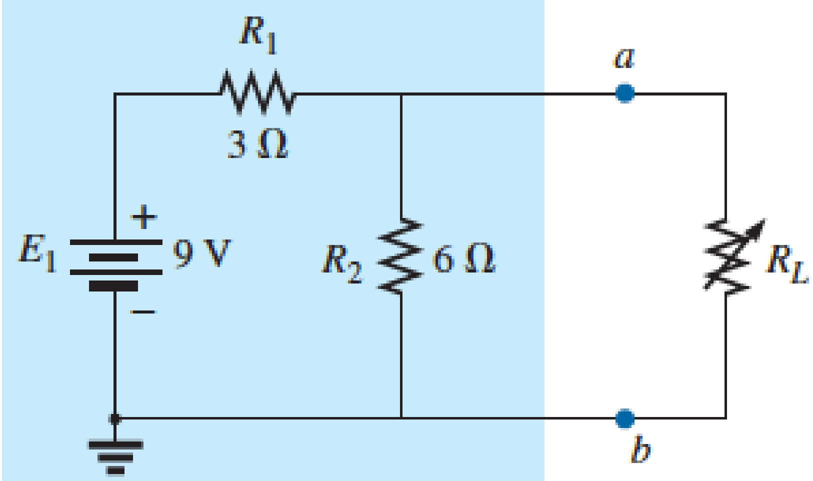


$$R_{Th} = R_1 \| R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = 2\ \Omega$$

Example (1)

Steps 4:





$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

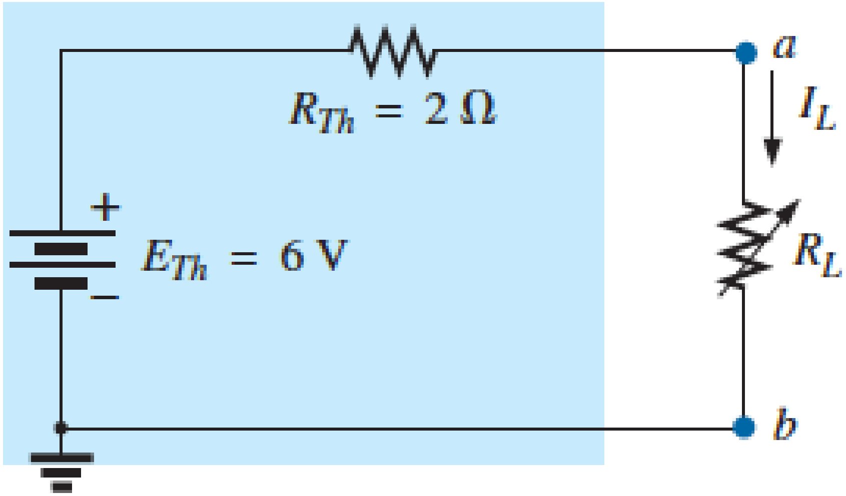
Steps 5:

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$R_L = 2\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

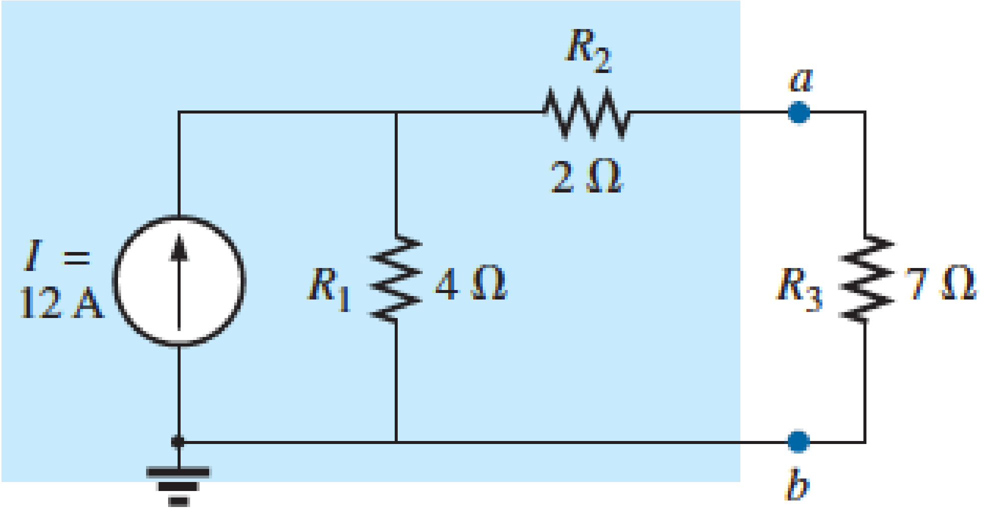
$R_L = 10\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$R_L = 100\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.06\ A}$$



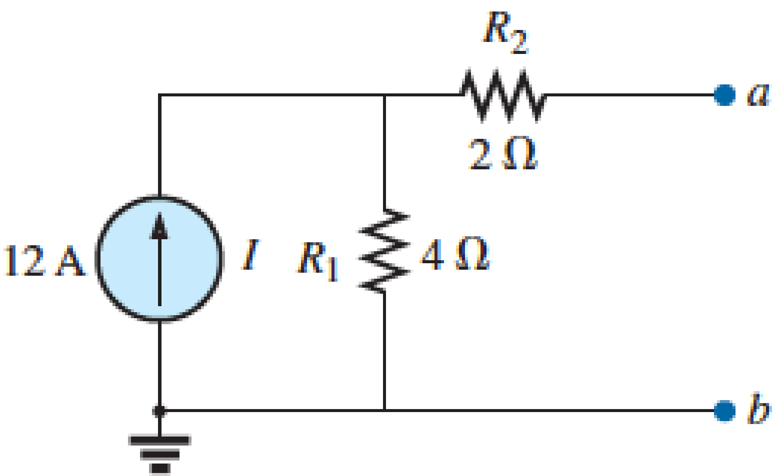
Example (2)

Find the Thévenin equivalent circuit for the network in the shaded area of the network .

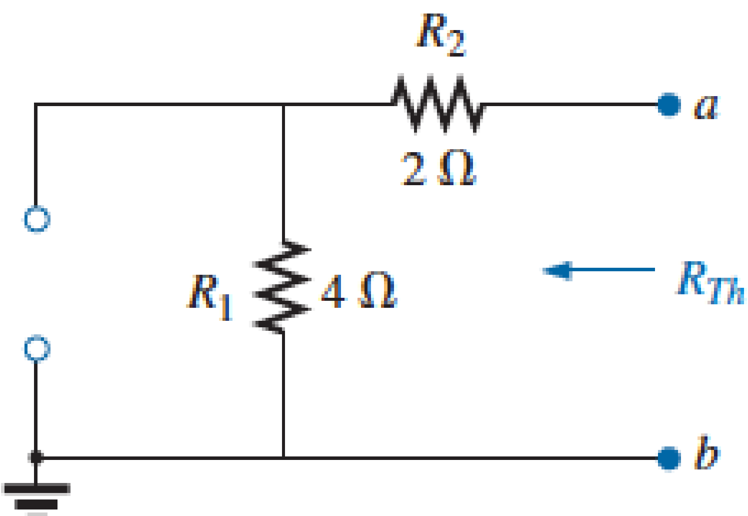


Solution:

Steps 1 and 2:



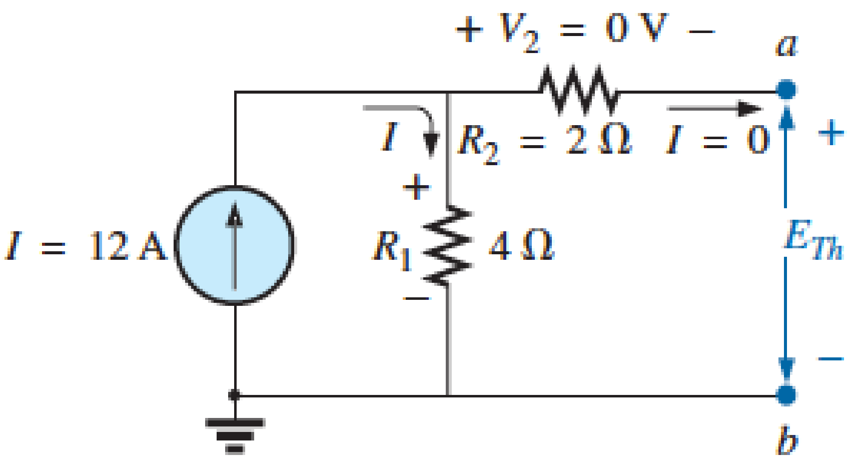
Steps 3:

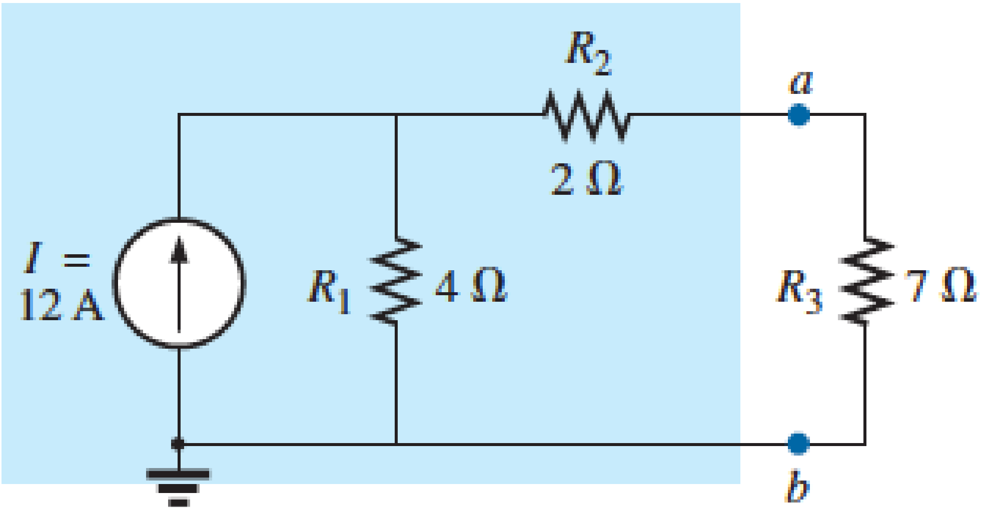


$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

Example (2)

Steps 4:

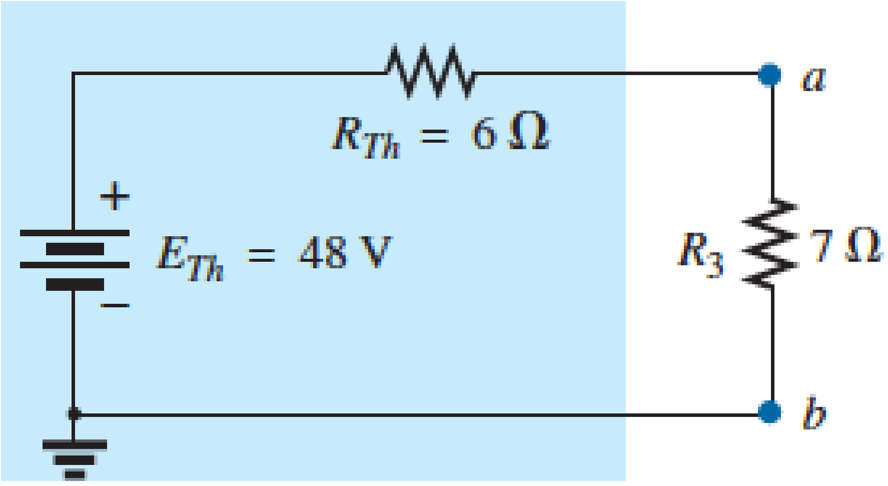




$$V_2 = I_2R_2 = (0)R_2 = 0\text{ V}$$

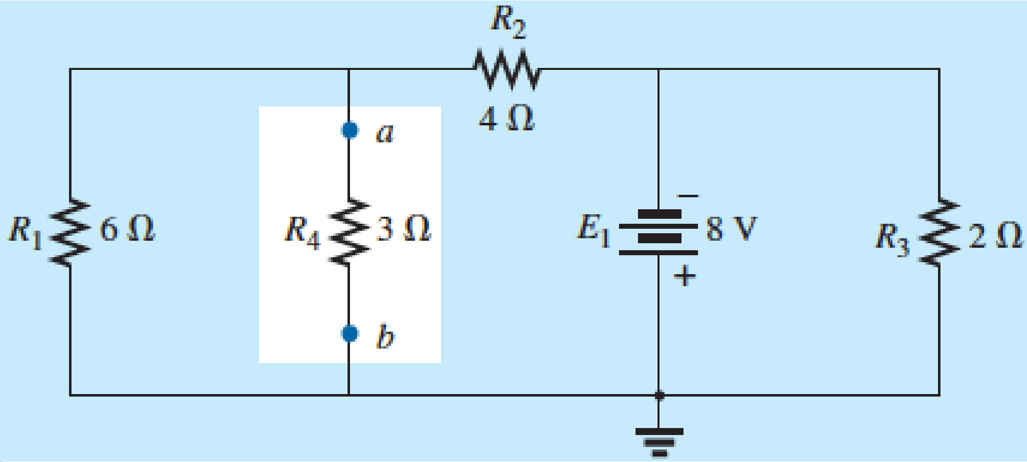
$$E_{Th} = V_1 = I_1R_1 = IR_1 = (12\text{ A})(4\ \Omega) = \mathbf{48\ V}$$

Steps 5:



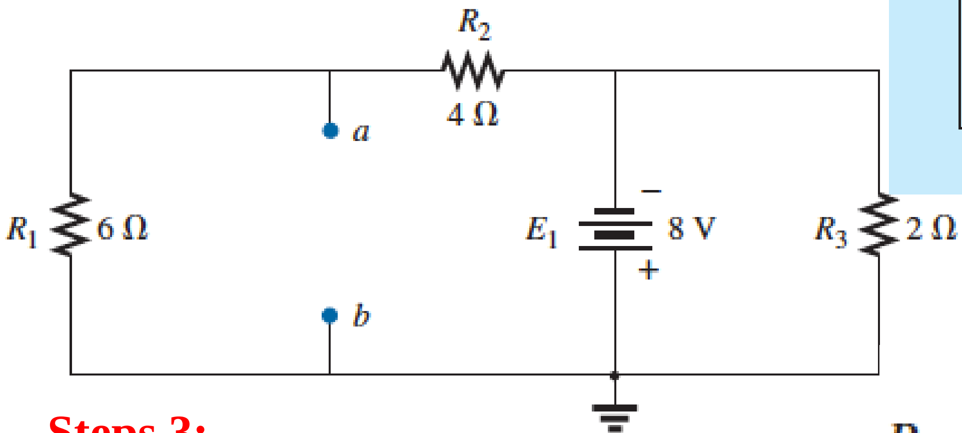
Example (3)

Find the Thévenin equivalent circuit for the network in the shaded area of the network .



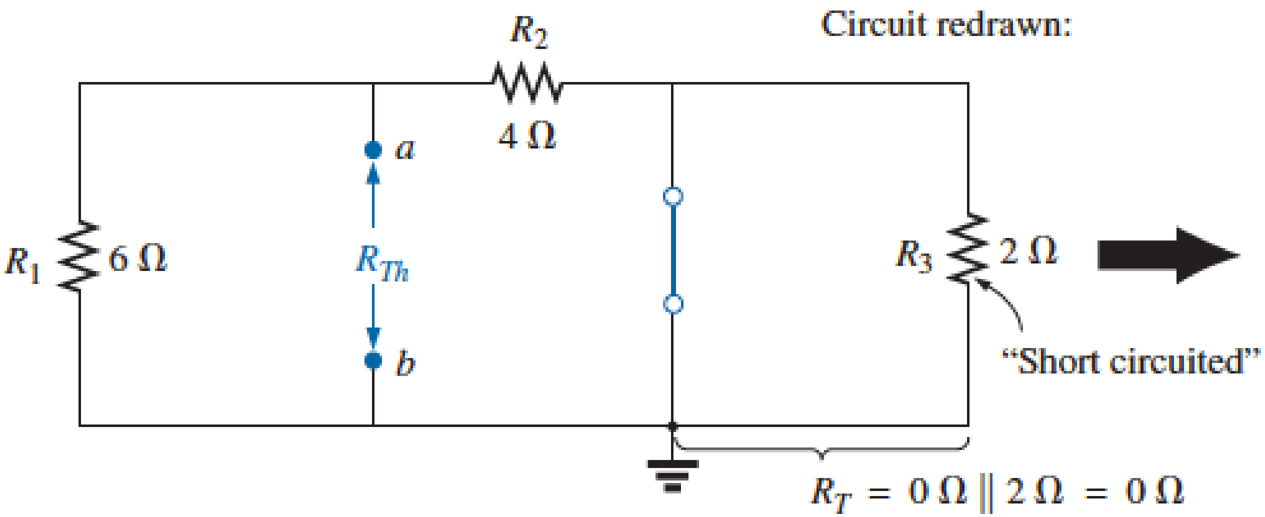
Solution:

Steps 1 and 2:

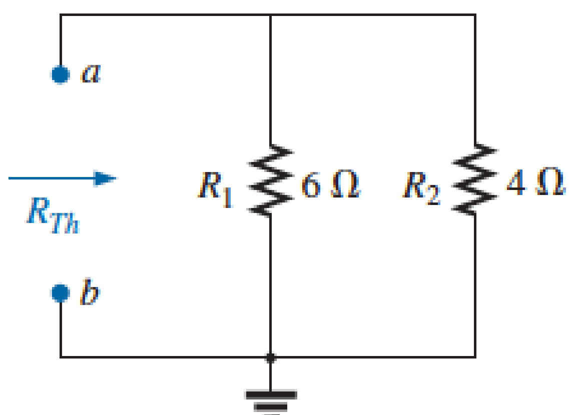


Steps 3:

$$R_{Th} = R_1 \| R_2 = \frac{(6\,\Omega)(4\,\Omega)}{6\,\Omega + 4\,\Omega} = \frac{24\,\Omega}{10} = \mathbf{2.4\,\Omega}$$

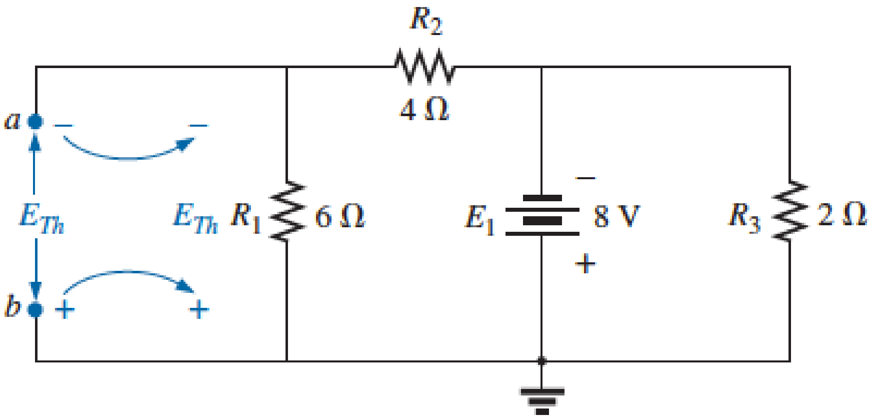


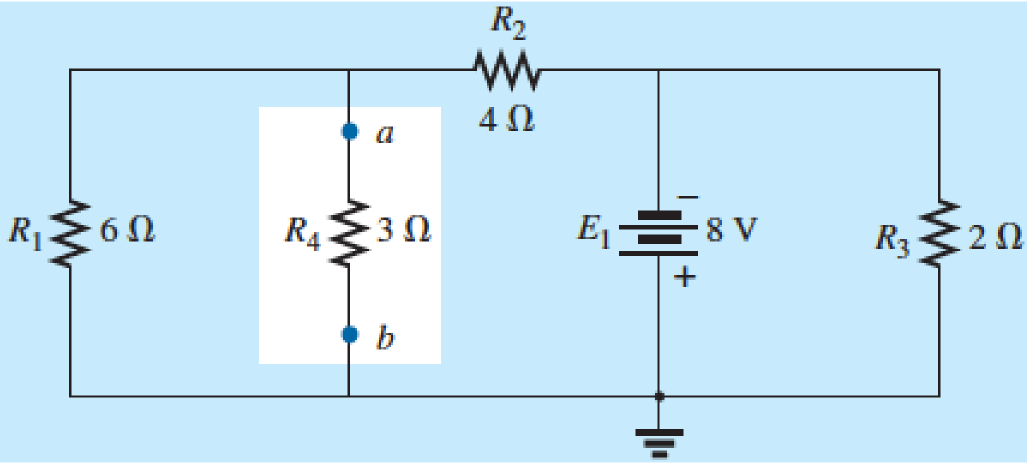
Circuit redrawn:

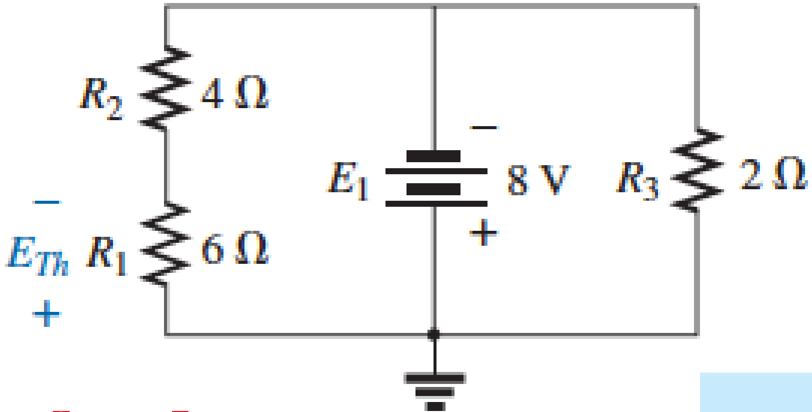


Example (3)

Steps 4:

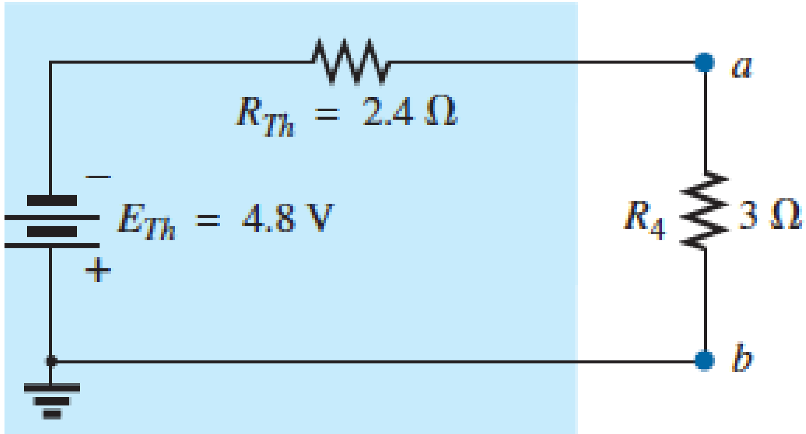






$$E_{Th} = \frac{R_1 E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ \text{V})}{6\ \Omega + 4\ \Omega} = \frac{48\ \text{V}}{10} = \mathbf{4.8\ V}$$

Steps 5:

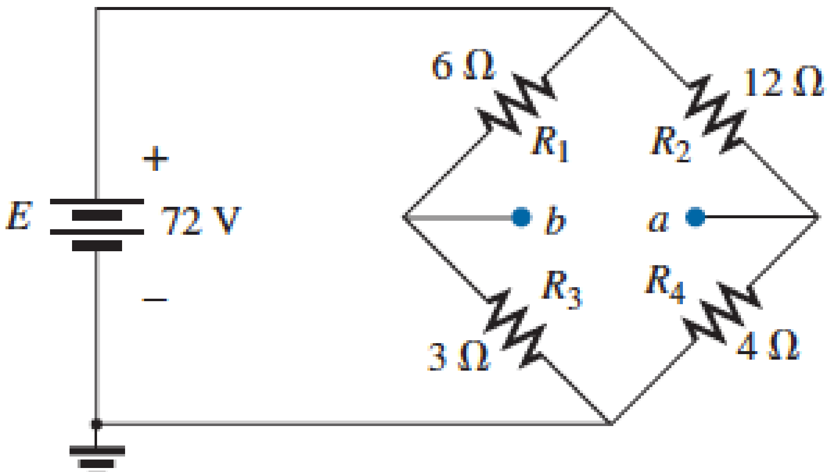


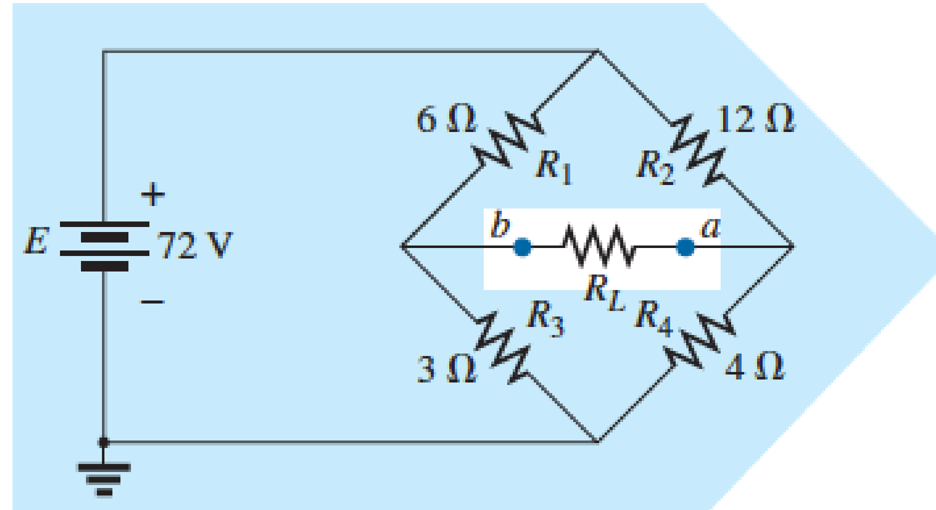
Example (4)

Find the Thévenin equivalent circuit for the network in the shaded area of the network .

Solution:

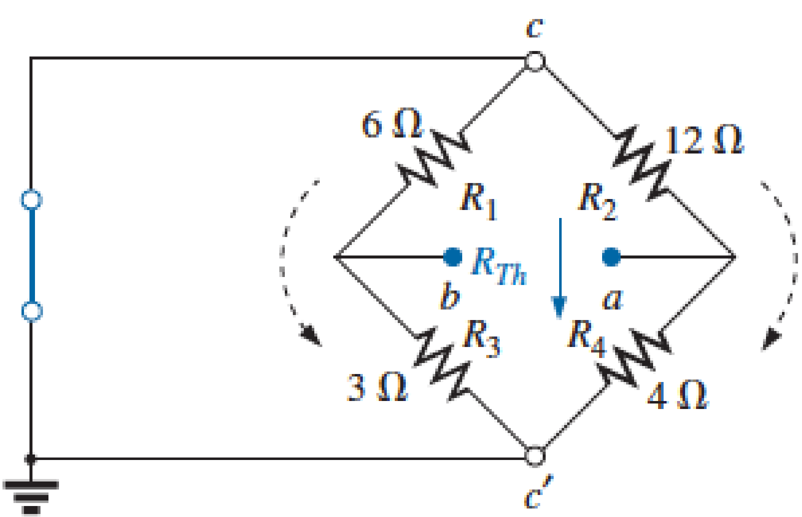
Steps 1 and 2:

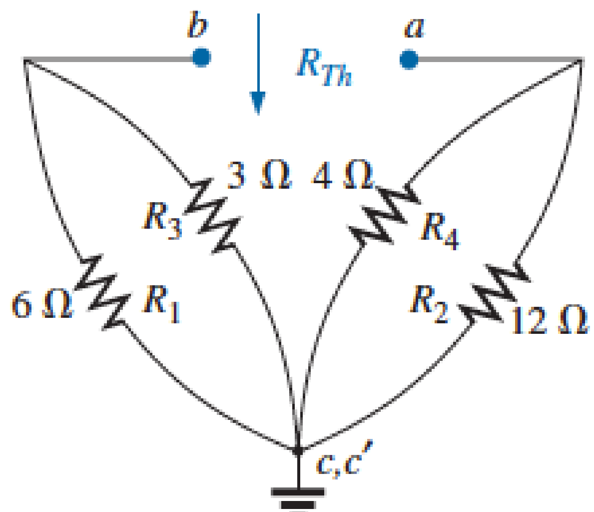




$$R_{Th} = R_{a-b} = R_1 \parallel R_3 + R_2 \parallel R_4$$
$$= 6\ \Omega \parallel 3\ \Omega + 4\ \Omega \parallel 12\ \Omega$$
$$= 2\ \Omega + 3\ \Omega = 5\ \Omega$$

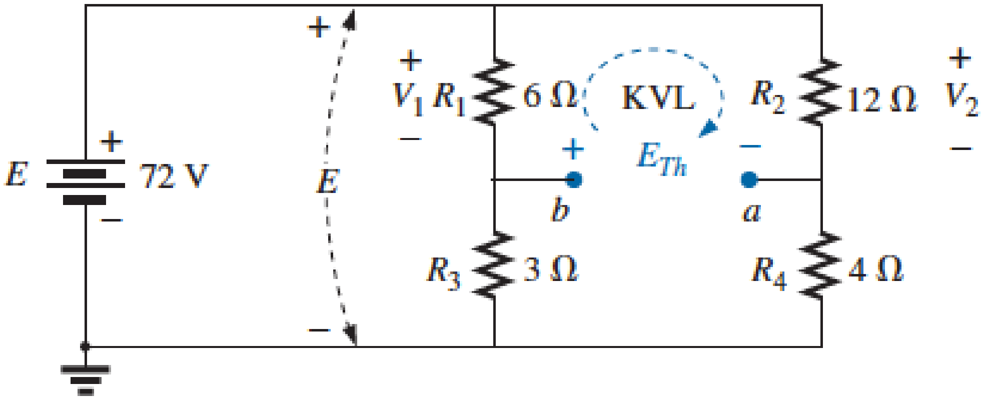
Steps 3:

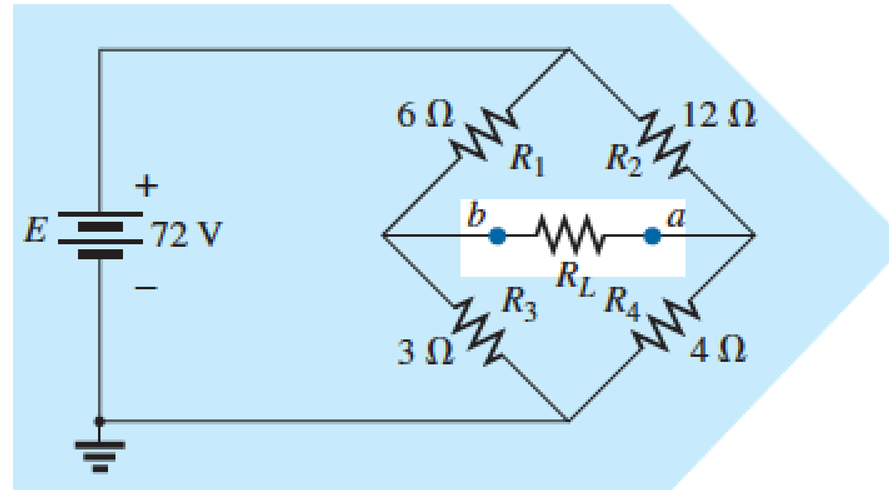




Example (4)

Steps 4:





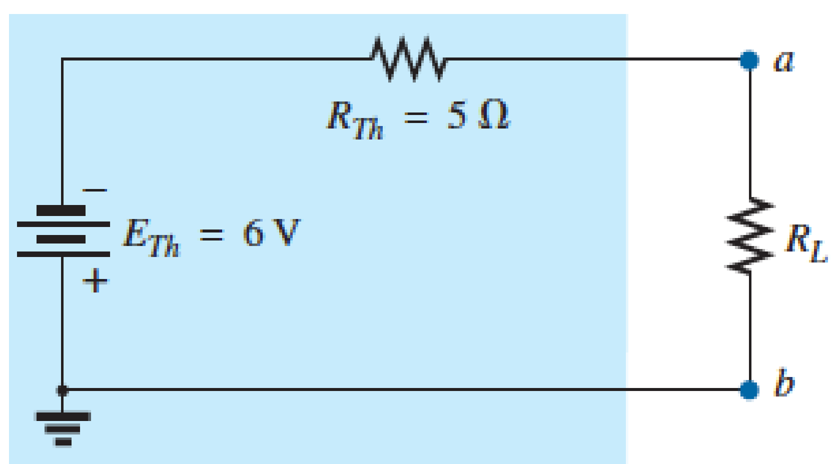
$$V_1 = \frac{R_1E}{R_1 + R_3} = \frac{(6\ \Omega)(72\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{432\ \text{V}}{9} = 48\ \text{V}$$

$$V_2 = \frac{R_2E}{R_2 + R_4} = \frac{(12\ \Omega)(72\ \text{V})}{12\ \Omega + 4\ \Omega} = \frac{864\ \text{V}}{16} = 54\ \text{V}$$

$$\Sigma_{\circlearrowright} V = +E_{Th} + V_1 - V_2 = 0$$

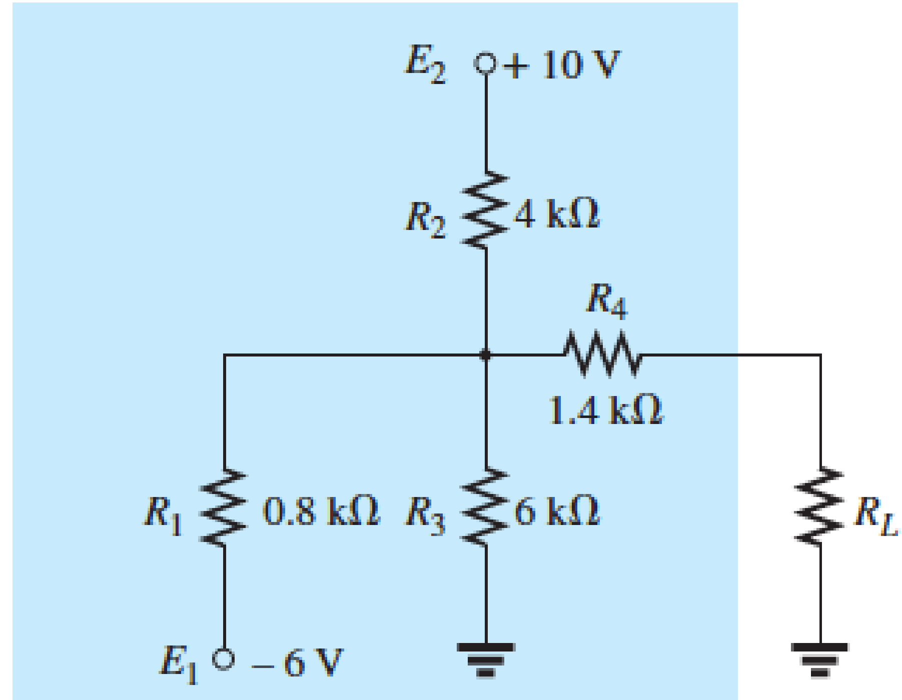
$$E_{Th} = V_2 - V_1 = 54\ \text{V} - 48\ \text{V} = \mathbf{6\ V}$$

Steps 5:



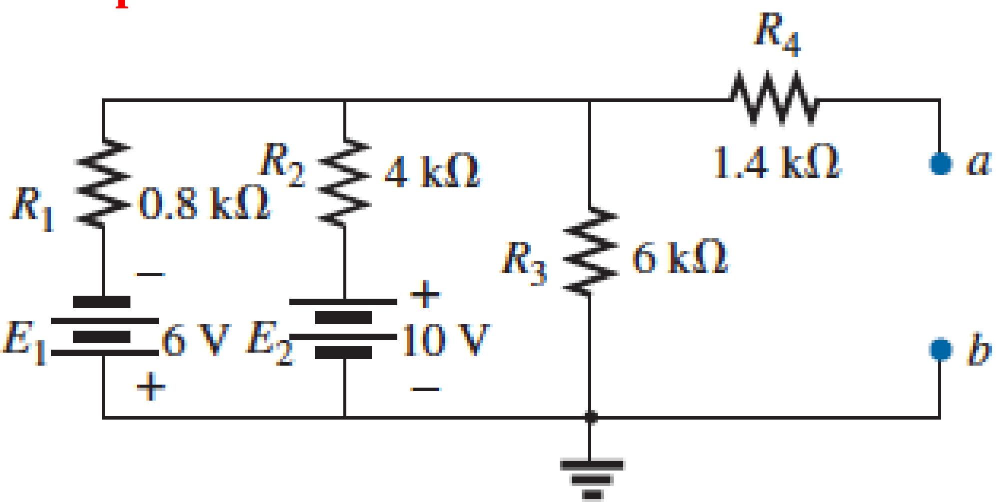
Example (5) (Two sources)

Find the Thévenin equivalent circuit for the network in the shaded area of the network .

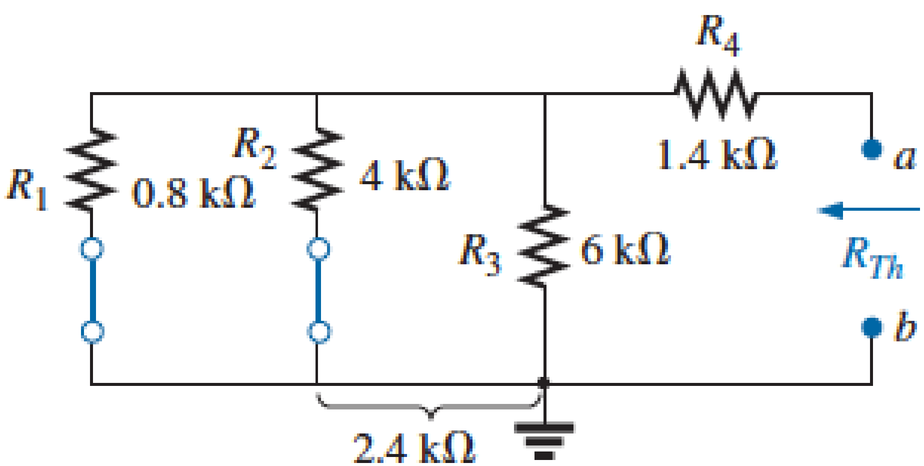


Solution:

Steps 1 and 2:



Steps 3:



$$
\begin{aligned}
R_{Th} &= R_4 + R_1 \| R_2 \| R_3 \\
&= 1.4\,\text{k}\Omega + 0.8\,\text{k}\Omega \| 4\,\text{k}\Omega \| 6\,\text{k}\Omega \\
&= 1.4\,\text{k}\Omega + 0.8\,\text{k}\Omega \| 2.4\,\text{k}\Omega \\
&= 1.4\,\text{k}\Omega + 0.6\,\text{k}\Omega \\
&= \mathbf{2\,k\Omega}
\end{aligned}
$$

Example (5)

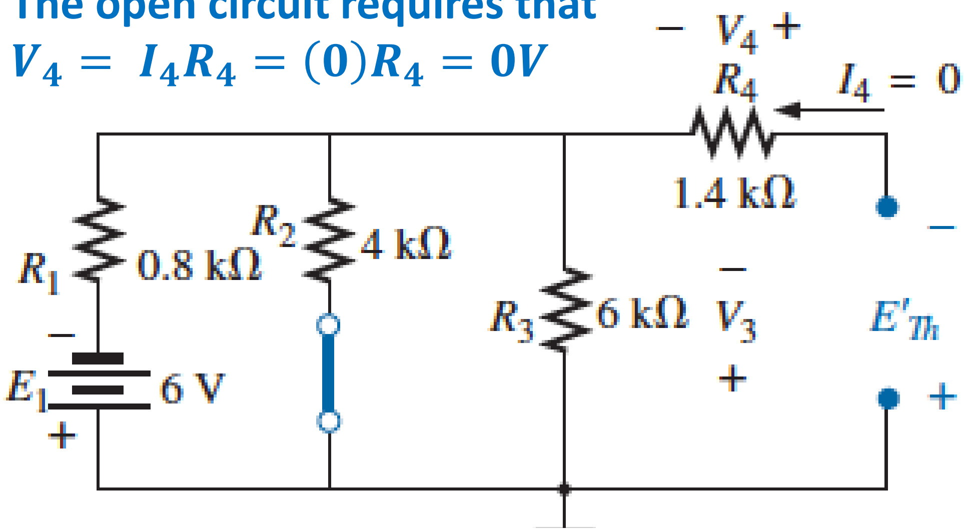
Steps 4:

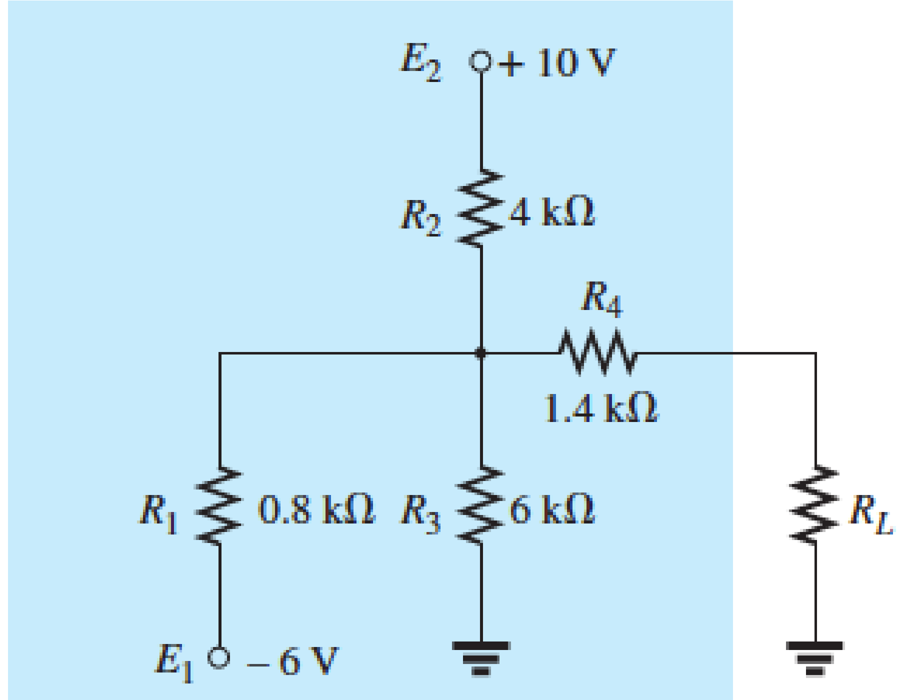
Applying superposition:

1) $E_2 = 0$

The open circuit requires that

$V_4 = I_4 R_4 = (0)R_4 = 0V$





$$E'_{Th} = V_3$$

$$R'_T = R_2 \| R_3 = 4\text{ k}\Omega \| 6\text{ k}\Omega = 2.4\text{ k}\Omega$$

Applying the voltage divider rule,

$$V_3 = \frac{R'_T E_1}{R_T} = \frac{(2.4\text{ k}\Omega)(6\text{ V})}{2.4\text{ k}\Omega + 0.8\text{ k}\Omega} = \frac{14.4\text{ V}}{3.2} = 4.5\text{ V}$$

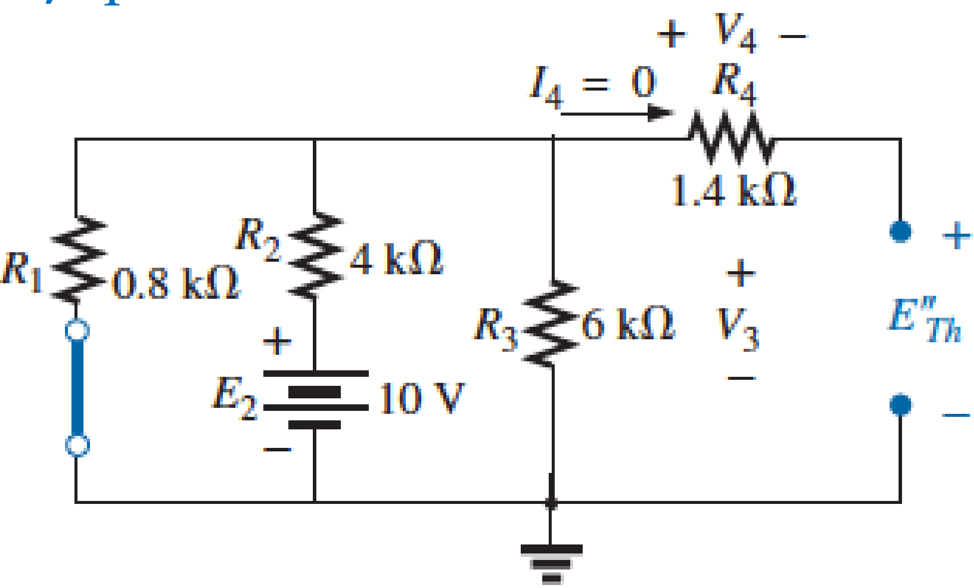
$$E'_{Th} = V_3 = 4.5\text{ V}$$

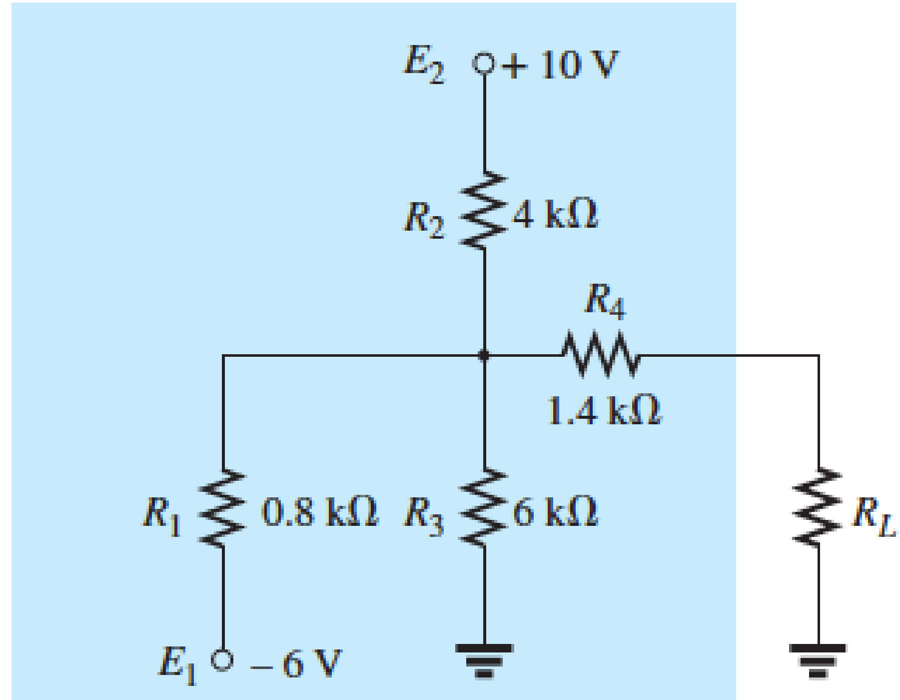
Example (5)

Steps 4:

Applying superposition:

2) $E_1 = 0$





$$E''_{Th} = V_3$$

$$R'_T = R_1 \| R_3 = 0.8\text{ k}\Omega \| 6\text{ k}\Omega = 0.706\text{ k}\Omega$$

$$V_3 = \frac{R'_T E_2}{R_T + R_2} = \frac{(0.706\text{ k}\Omega)(10\text{ V})}{0.706\text{ k}\Omega + 4\text{ k}\Omega} = \frac{7.06\text{ V}}{4.706} = 1.5\text{ V}$$

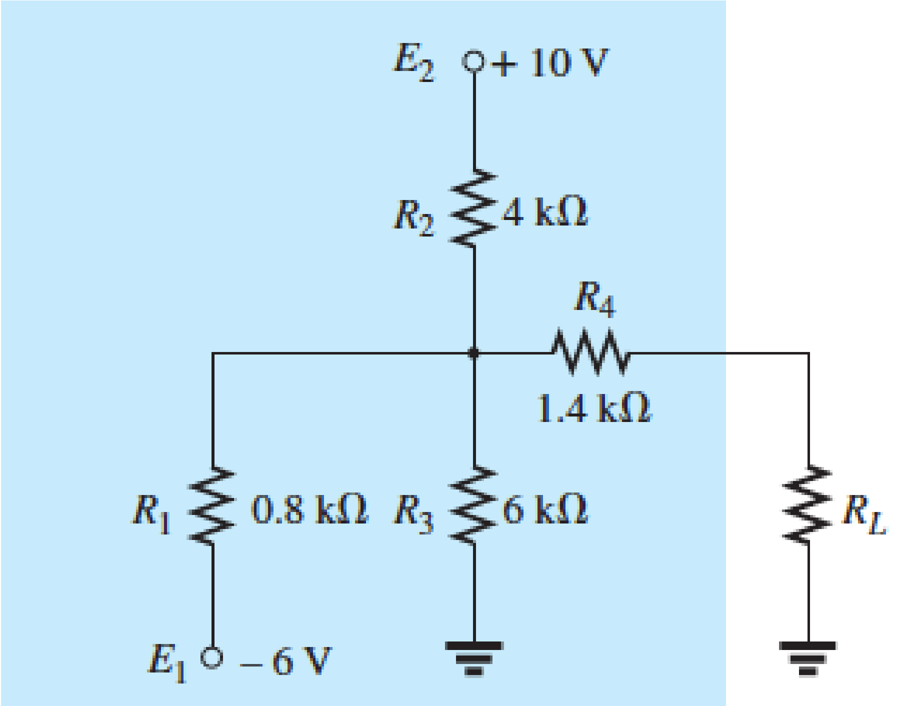
$$E''_{Th} = V_3 = 1.5\text{ V}$$

Example (5)

Steps 4:

Since E'_{Th} and E''_{Th} have opposite polarities,

$$\begin{aligned} E_{Th} &= E'_{Th} - E''_{Th} \\ &= 4.5\text{ V} - 1.5\text{ V} \\ &= 3\text{ V} \qquad \text{(polarity of } E'_{Th}\text{)} \end{aligned}$$



Steps 5: