University of Diyala

College of Engineering

Department of Power and Electrical Machines Engineering

Academic Year 2025-2026

Second Year – Bachelor (BSc) – EPE 203

Fall Semester

EPE 203 Electric Circuits Analysis I

Lecture 2

Assist. Lect. Ibrahim I. Ibrahim

Lecture 2

Balanced $\mathrm{Y} - \mathrm{Y}$, $\mathrm{Y} - \Delta$, $\Delta - \Delta$, and $\Delta - \mathrm{Y}$ connections.

Assist. Lect. Ibrahim I. Ibrahim

The three-phase load can be either $Y -$ connected or $\Delta -$ connected, depending on the end application. **Figure 1**, shows the two type of three-phase load.

The neutral line in **Figure 1(a)** may or may not be there, depending on whether the system is four- or three-wire.

$Y -$ connected or $\Delta -$ connected load is said to be unbalanced if the phase impedances are not equal in magnitude or phase.

A **balanced load** is one in which the phase impedances are equal in magnitude and in phase.

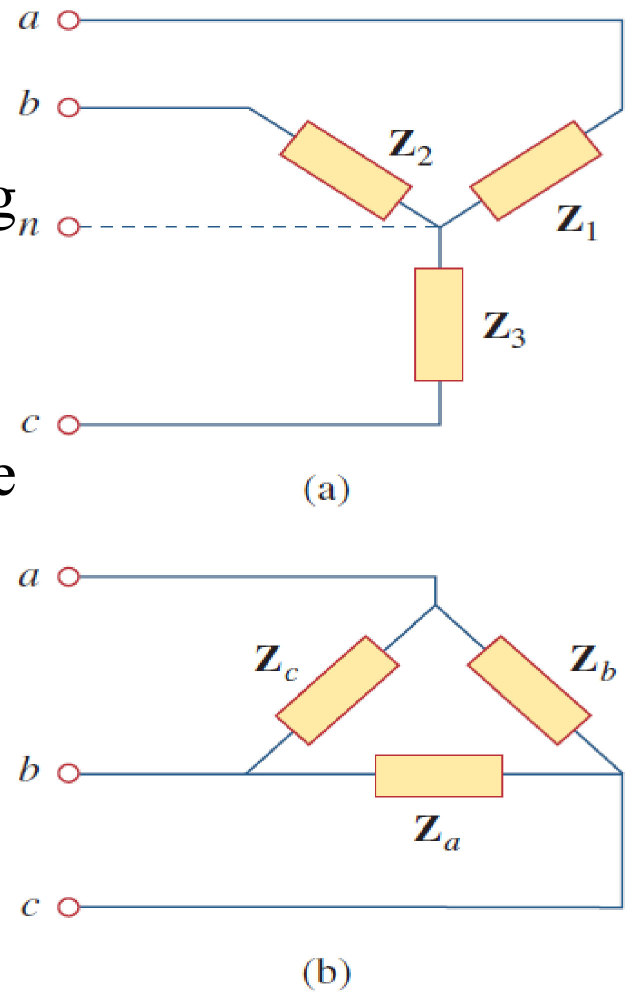


Figure 1: Three-phase load configurations

For a balanced $\mathrm{Y}-$connected load, we have: $\boldsymbol{Z_Y = Z_1 = Z_2 = Z_3}$ where $\boldsymbol{Z_Y}$ is the load impedance per phase.

For a balanced $\Delta-$connected load, we have: $\boldsymbol{Z_\Delta = Z_a = Z_b = Z_c}$ where $\boldsymbol{Z_\Delta}$ is the load impedance per phase.

Now, the $\mathrm{Y}-$connected load can be transformed into a $\Delta-$connected load or vice versa via the following: $\boldsymbol{Z_\Delta = 3Z_Y}$ or $\boldsymbol{Z_Y = \frac{Z_\Delta}{3}}$

Since both the three-phase source and the three-phase load can be either Y or Δ connected, we have four possible connections:

1) $\mathrm{Y}-\mathrm{Y}$ connection.
2) $\mathrm{Y}-\Delta$ connection.
3) $\Delta-\Delta$ connection.
4) $\Delta-\mathrm{Y}$ connection.

Remark

A balanced $\Delta -$ connected load is generally more common than a balanced $\mathrm{Y} -$ connected load, since it allows loads to be added or removed from individual phases with relative ease. In contrast, this process is difficult in a $\mathrm{Y} -$ connected system, especially when the neutral point is not available. However, $\Delta -$ connected sources are rarely used in practice, as even small imbalances in the three-phase voltages can produce circulating currents within the delta loop.

Part 1:

Y – Y connection

A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.

Consider the balanced four-wire Y-Y system of **Figure 2**. Assume a balanced load so that load impedances are equal.

Although the impedance $\boldsymbol{Z_Y}$ is the total load impedance per phase, it may also be regarded as the sum of the source impedance $\boldsymbol{Z_s}$ line impedance $\boldsymbol{Z_l}$ and load Impedance $\boldsymbol{Z_L}$ for each phase, since these impedances are in series.

$$Z_Y = Z_s + Z_l + Z_L$$

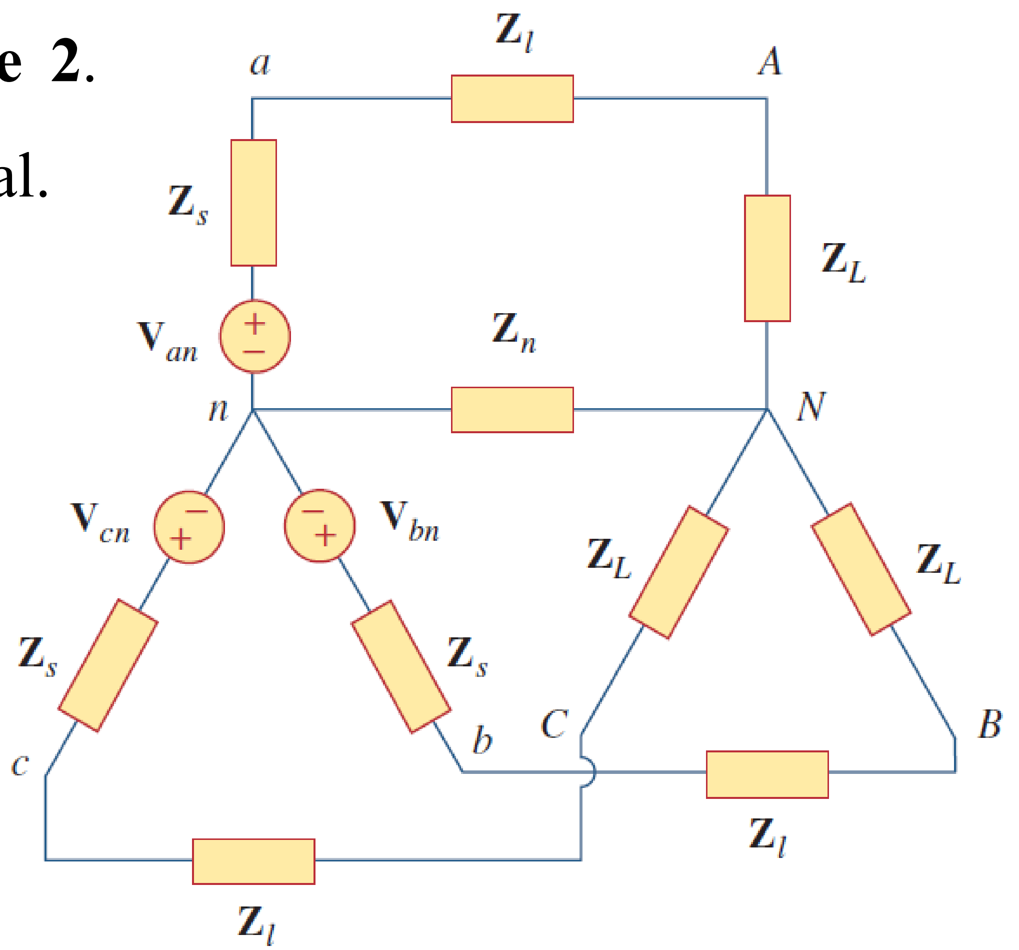


Figure 2: A balanced Y-Y system.

Assist. Lect. Ibrahim I. Ibrahim

It's significant to mention that Z_s and Z_l are often too small in comparison to Z_L. Thus, we can assume that $Z_Y = Z_L$ if source or line impedance is given. In any event, by lumping the impedances together, the Y-Y system in **Figure 2** can be simplified to that shown in **Figure 3**.

Assuming the abc sequence, the phase voltages (or line-to neutral voltages) are

$$V_{an} = V_P\angle 0^\circ$$

$$V_{bn} = V_P\angle -120^\circ$$

$$V_{cn} = V_P\angle +120^\circ$$

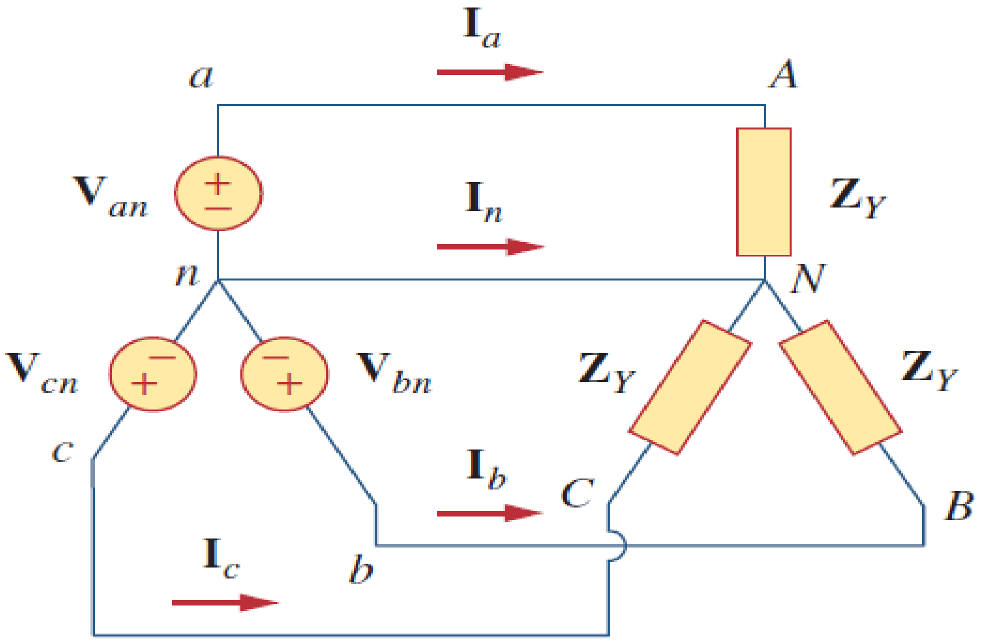


Figure 3: A balanced Y-Y system.

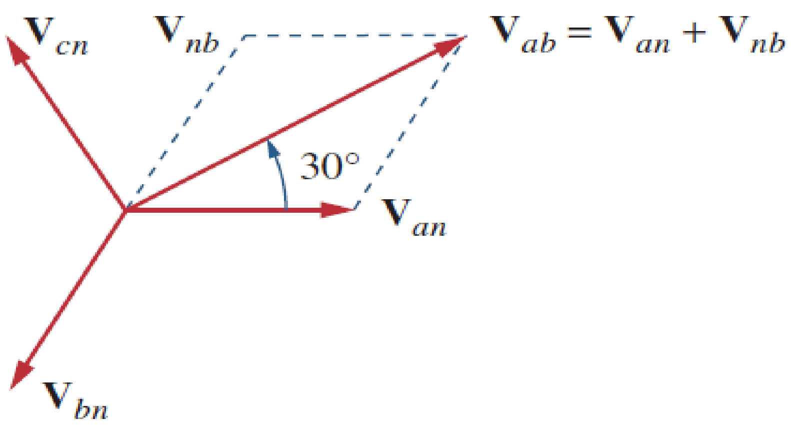
The line-to-line voltages or simply line voltages V_{ab}, V_{bc} and V_{ca} are related to the phase voltages.

$$\boxed{V_{ab} = V_{an} - V_{bn} = V_P\angle 0^{\circ} - V_P\angle -120^{\circ} \Longrightarrow V_{ab} = V_P\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_P\angle 30^{\circ}}$$

$$\boxed{V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_P\angle -90^{\circ}} \qquad \boxed{V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_P\angle -210^{\circ}}$$

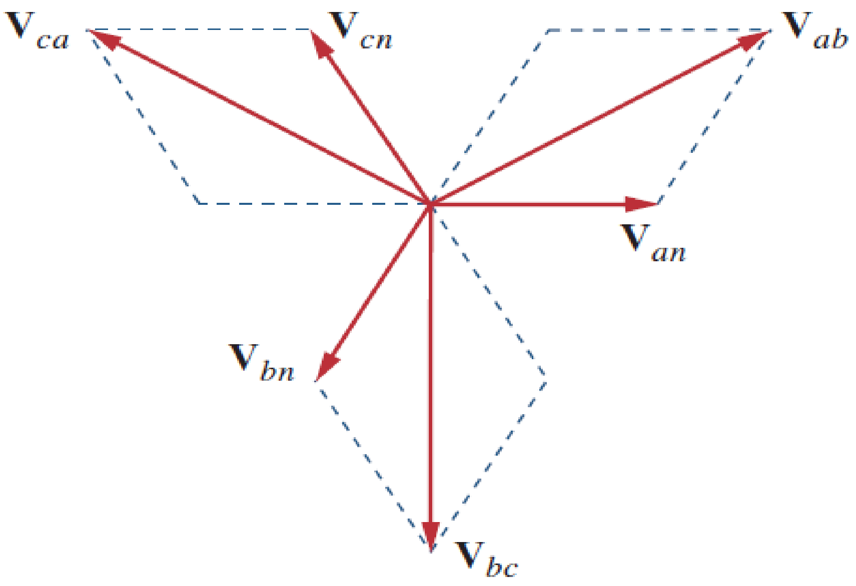
Thus, the magnitude of the line voltages V_L is times the magnitude of the phase voltages V_P or mathematically it can be expressed as: $V_L = \sqrt{3}V_P$

$$\boxed{V_P = |V_{an}| = |V_{bn}| = |V_{cn}|} \qquad \boxed{V_L = |V_{ab}| = |V_{bc}| = |V_{ca}|}$$



(a)

The phasor diagram on the left, part (a) shows that the line voltages lead their corresponding phase voltages by 30° and it shows how to determine V_{ab} from the phase voltages.



(b)

While part (b) shows the same for the three-line voltages. Notice that V_{ab} leads V_{bc} by 120° and V_{bc} leads V_{ca} by 120°, so that the line voltages sum up to zero as do the phase voltages.

Now, we obtain the line currents as follows:

$$\boxed{I_a = \frac{V_{an}}{Z_Y}}$$

$$\boxed{I_b = \frac{V_{bn}}{Z_Y} = \frac{V_{an}\angle -120^{\circ}}{Z_Y} = I_a\angle -120^{\circ}}$$

$$\boxed{I_c = \frac{V_{cn}}{Z_Y} = \frac{V_{an}\angle -240^{\circ}}{Z_Y} = I_a\angle -240^{\circ}}$$

We can readily infer that the line currents add up to zero, $\boxed{I_a + I_b + I_c = 0}$

$\boxed{I_n = -(I_a + I_b + I_c) = 0}$ or $\boxed{\mathrm{V_{nN} = Z_N I_N = 0}}$

The voltage across the neutral wire is zero. The neutral line can thus, be removed without affecting the system.

Ex1: Calculate the line currents in the three-wire Y-Y system shown below.

First, let's determine the impedance Z_Y

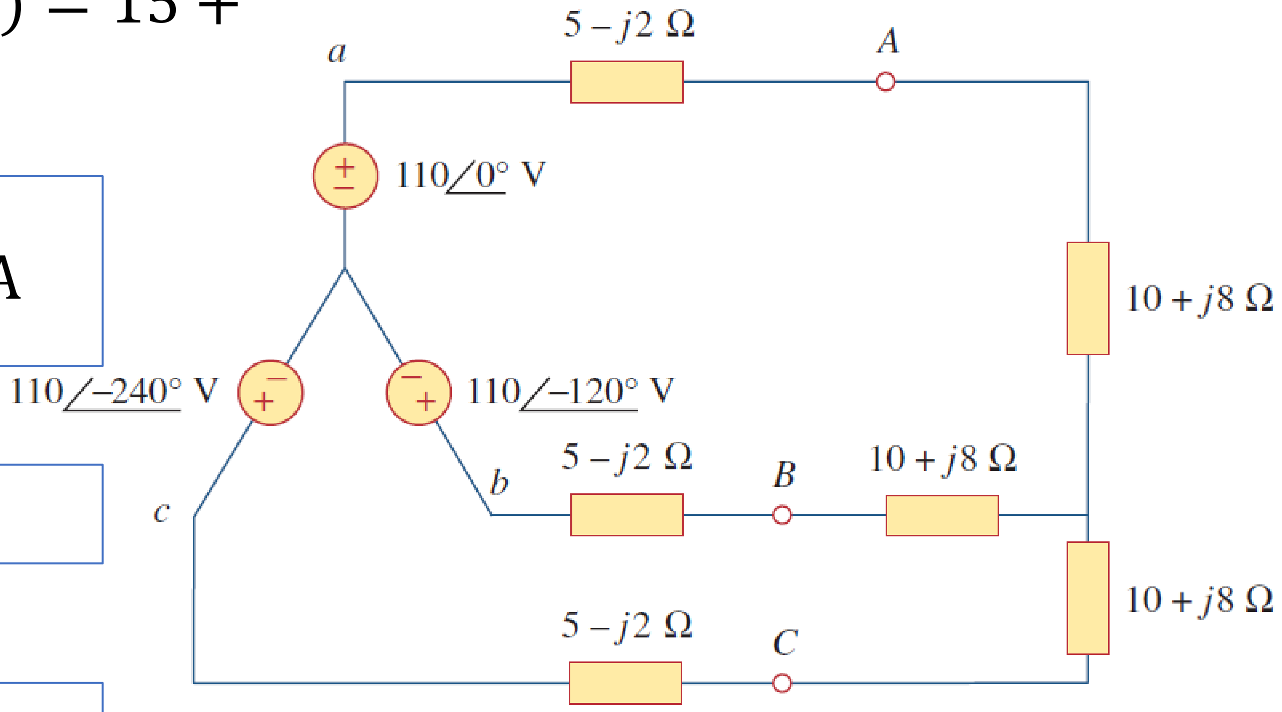
$$Z_Y = Z_l + Z_L \implies Z_Y = (5 - j2) + (10 + j8) = 15 + j6\ \Omega$$

$$15 + j6\ \Omega = 16.155\angle 21.8^\circ\ \Omega$$

$$I_a = \frac{V_{an}}{Z_Y} = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \mathrm{A}$$

$$I_b = I_a\angle -120^\circ = 6.81\angle -141.8^\circ \mathrm{A}$$

$$I_c = I_a\angle -240^\circ = 6.81\angle -261.8^\circ \mathrm{A}$$



Part 2:

$Y - \Delta$ connection

A balanced $Y - \Delta$ system is a three-phase system with a balanced Y-connected source and a balanced Δ - connected load.

The balanced Y- Δ system is shown in **Figure 3**, where the source is Y and the load is Δ. Of course, no neutral connection from source to load for this case. Assuming the abc sequence, the phase voltages are again $V_{an} = V_P\angle 0^\circ$ $V_{bn} = V_P\angle -120^\circ$ $V_{cn} = V_P\angle +120^\circ$

The line voltages are $V_{ab} = \sqrt{3}V_P\angle 30^\circ = V_{AB}$

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_P\angle -90^\circ = V_{BC}$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_P\angle -150^\circ = V_{CA}$$

From these voltages, we can obtain the phase currents as

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} \qquad I_{BC} = \frac{V_{BC}}{Z_\Delta} \qquad I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

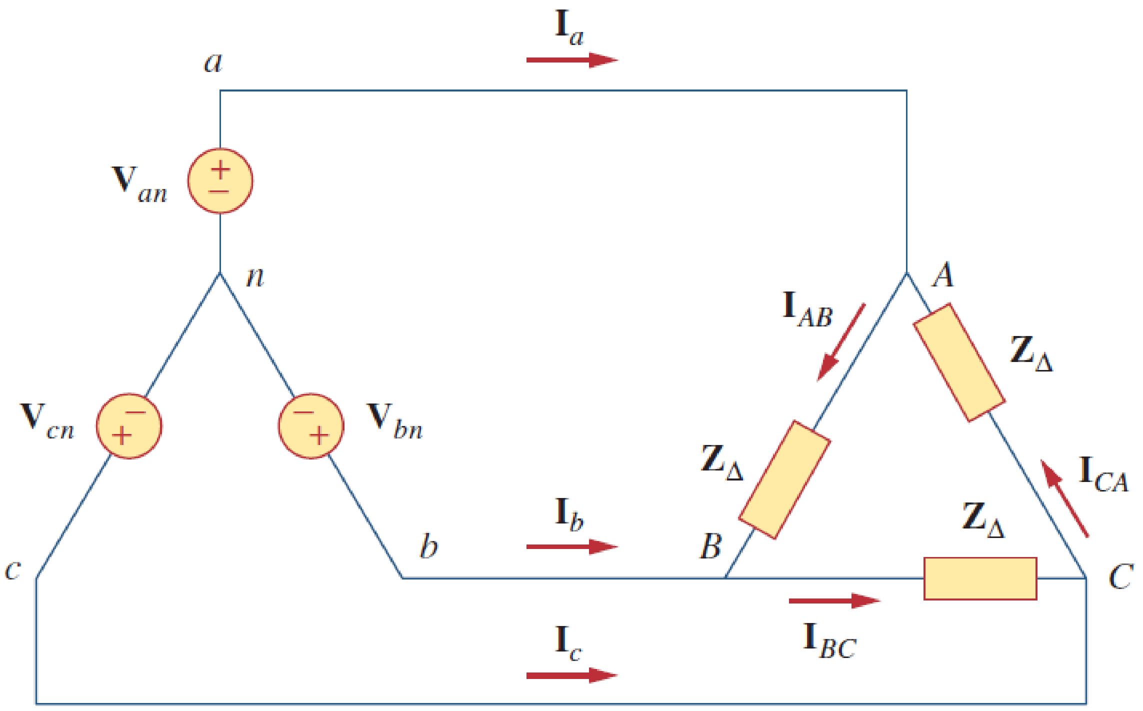


Figure 3: A balanced Δ-Y system.

Another way to get these phase currents is to apply KVL.

$$Z_\Delta I_{AB} + V_{bn} - V_{an} = 0 \Longrightarrow I_{AB} = \frac{V_{an} - V_{bn}}{Z_\Delta} = \frac{V_{AB}}{Z_\Delta}$$

The line currents are obtained from the phase currents by applying KCL at nodes A, B, and C. Thus,

$$\boxed{I_a = I_{AB} - I_{CA}} \qquad \boxed{I_b = I_{BC} - I_{AB}} \qquad \boxed{I_c = I_{CA} - I_{BC}}$$

Since $I_{CA} = I_{AB}\angle -240^\circ$

$$\boxed{I_a = I_{AB} - I_{CA} = I_{AB}\left(1 - 1\angle -240^\circ\right) = I_{AB}(1 + 0.5 - j0.866) \Longrightarrow I_a = I_{AB}\sqrt{3}\angle -30^\circ}$$

Thus, the magnitude of the line current I_L is $\sqrt{3}$ times the magnitude of the phase current I_P

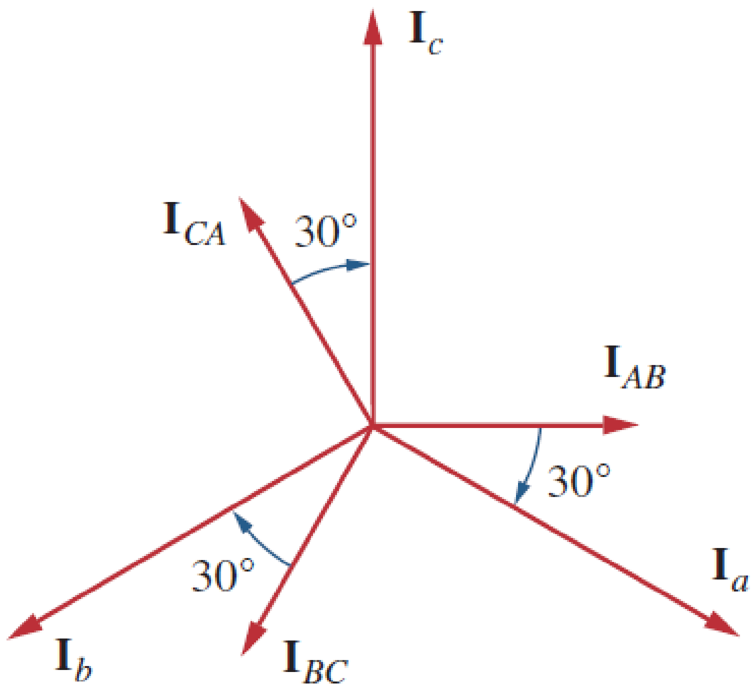
$$I_L = \sqrt{3} I_P$$

$$\boxed{I_P = |I_{AB}| = |I_{BC}| = |I_{CA}|} \qquad \boxed{I_L = |I_a| = |I_b| = |I_c|}$$

An alternative way of analyzing the $Y-\Delta$ circuit is to transform the Δ-connected load to an equivalent Y-connected load via using the transformation formula

$$Z_Y = \frac{Z_\Delta}{3}$$

The phasor diagram on the right, illustrating the relationship between phase and line currents.



This is perhaps the most practical three-phase system, as the three-phase sources are usually Y-connected while the three-phase loads are usually Δ -connected.

Ex2: A balanced abc-sequence Y-connected source with $V_{an} = 100\angle 10^{\circ} V$ is connected to a Δ −connected balanced load $(8 + j4)\Omega$ per phase. Calculate the phase and line currents.

$\boldsymbol{Z_{\Delta}} = 8 + \mathrm{j}4 = 8.944\angle 26.57^{\circ}\boldsymbol{\Omega}$ $V_{ab} = \sqrt{3}V_{P}\angle 30^{\circ} = V_{AB} = 100\sqrt{3}\angle 30^{\circ} + 10^{\circ} = 173.2\angle 40^{\circ}V$

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{173.2\angle 40^{\circ}}{\mathbf{8.944\angle 26.57^{\circ}}} = 19.36\angle 13.43^{\circ} A$$

$$I_{BC} = I_{AB}\angle -120^{\circ} = 19.36\angle -106.57^{\circ}\ A$$

$$I_{CA} = I_{AB}\angle +120^{\circ} = 19.36\angle 133.43^{\circ} A$$

$$I_{a} = I_{AB}\sqrt{3}\angle -30^{\circ} = \sqrt{3}\ 19.36\angle 13.43^{\circ} - 30^{\circ} = 33.53\angle -16.57^{\circ}\ A$$

$$I_{\mathrm{b}} = I_{\mathrm{a}}\angle -120^{\circ} = 33.53\angle -136.57^{\circ}\ A$$

$$I_{\mathrm{c}} = I_{\mathrm{a}}\angle +120^{\circ} = 33.53\angle 103.43^{\circ} A$$

Part 3:

$\Delta - \Delta$ connection

A balanced $\Delta - \Delta$ system is a three-phase system with a balanced Δ -connected source and a balanced Δ - connected load.

The balanced Δ - Δ system is shown in **Figure 4**, where the source is Δ and the load is Δ. Assuming the abc sequence, the phase voltages are again

$$\boxed{V_{ab} = V_P \angle 0^\circ} \qquad \boxed{V_{bc} = V_P \angle -120^\circ} \qquad \boxed{V_{ca} = V_P \angle +120^\circ}$$

The line voltages are the same as the phase voltages. From **Figure 4**. assuming there is no line impedances, the phase voltages of the $\Delta -$ connected source are equal to the voltages across the impedances; that is,

$$\boxed{V_{ab} = V_{AB}} \qquad \boxed{V_{bc} = V_{BC}} \qquad \boxed{V_{ca} = V_{CA}}$$

Hence, the phase currents are

$$\boxed{I_{AB} = \frac{V_{AB}}{Z_\Delta}} \qquad \boxed{I_{BC} = \frac{V_{BC}}{Z_\Delta}} \qquad \boxed{I_{CA} = \frac{V_{CA}}{Z_\Delta}}$$

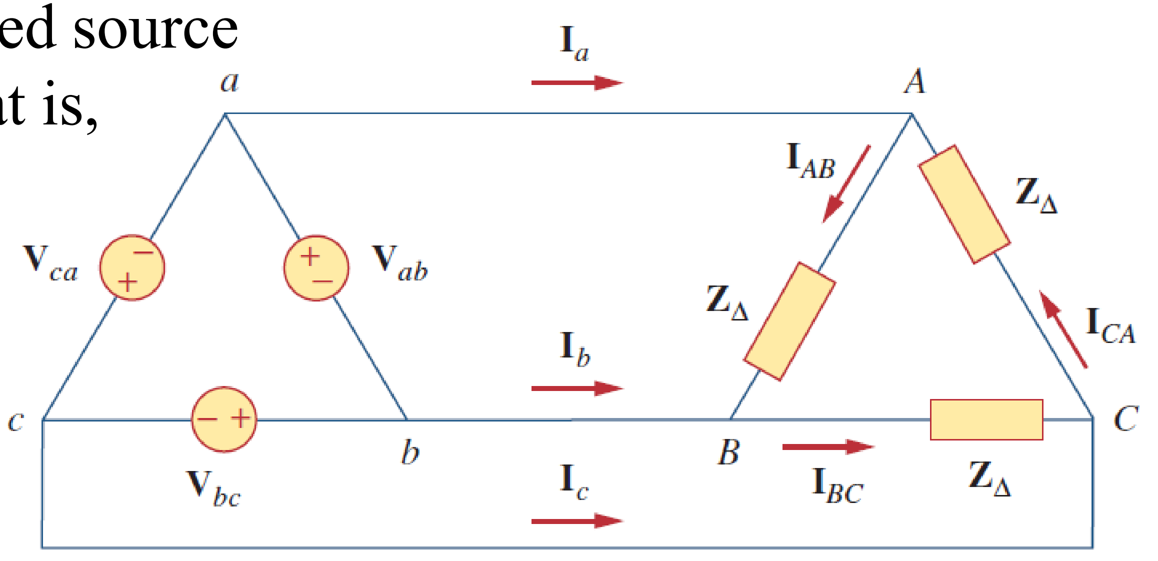


Figure 4: A balanced Δ- Δ system.

Since the load is $\Delta -$ connected just as in the previous part, line currents formulas derived there can be applied here.

Ex3: A balanced abc-sequence Δ-connected source with $V_{ab} = 330\angle 0^{\circ} V$ is connected to a Δ −connected balanced load $(20 - j15)\Omega$ per phase. Calculate the phase and line currents.

$$\boldsymbol{Z}_{\Delta} = 20 - j15 = 25\angle -36.87^{\circ}\ \boldsymbol{\Omega}$$

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{330\angle 0^{\circ}}{25\angle -36.87^{\circ}} = 13.2\angle 36.87^{\circ} A$$

$$I_{BC} = I_{AB}\angle{-120^{\circ}} = 13.2\angle -83.13^{\circ}\ A$$

$$I_{CA} = I_{AB}\angle{+120^{\circ}} = 13.2\angle 156.87^{\circ} A$$

$$I_{a} = I_{AB}\sqrt{3}\angle{-30^{\circ}} = (13.2\angle 36.87^{\circ})(\sqrt{3}\angle{-30^{\circ}}) = 22.86\ \angle 6.87^{\circ} A$$

$$I_{\mathrm{b}} = I_{\mathrm{a}}\angle{-120^{\circ}} = 22.86\ \angle -113.13^{\circ}\ A$$

$$I_{\mathrm{c}} = I_{\mathrm{a}}\angle{+120^{\circ}} = 22.86\ \angle 126.87^{\circ} A$$

Part 4:

$\Delta - \text{Y}$ connection

A balanced $\Delta - \text{Y}$ system is a three-phase system with a balanced Δ -connected source and a balanced Y - connected load.

The balanced Δ - Y system is shown in **Figure 5**, where the source is Δ and the load is Y. Assuming the abc sequence, the phase voltages are again

$\boxed{V_{ab} = V_P\angle 0^{\circ}}$ $\boxed{V_{bc} = V_P\angle -120^{\circ}}$ $\boxed{V_{ca} = V_P\angle +120^{\circ}}$

These are also the line voltages as well as the phase voltages.

We can obtain the line currents in many ways. One way is to apply KVL to loop aANBba in Figure 5.

$$Z_Y I_a - Z_Y I_b - V_{ab} = 0 \implies Z_Y(I_a - I_b) = V_{ab}$$

$$I_a - I_b = \frac{V_{ab}}{Z_Y} = \frac{V_P\angle 0^{\circ}}{Z_Y}$$

We know that I_b lags I_a by 120°

$$I_a - I_b = I_a(1 - 1\angle -120^0) = I_a\sqrt{3}\angle 30^{\circ}$$

$$\boxed{I_a = \frac{\frac{V_P}{\sqrt{3}\angle -30^{\circ}}}{Z_Y}}$$

$\boxed{I_b = I_a\angle -120^{\circ}}$ $\boxed{I_c = I_a\angle +120^{\circ}}$

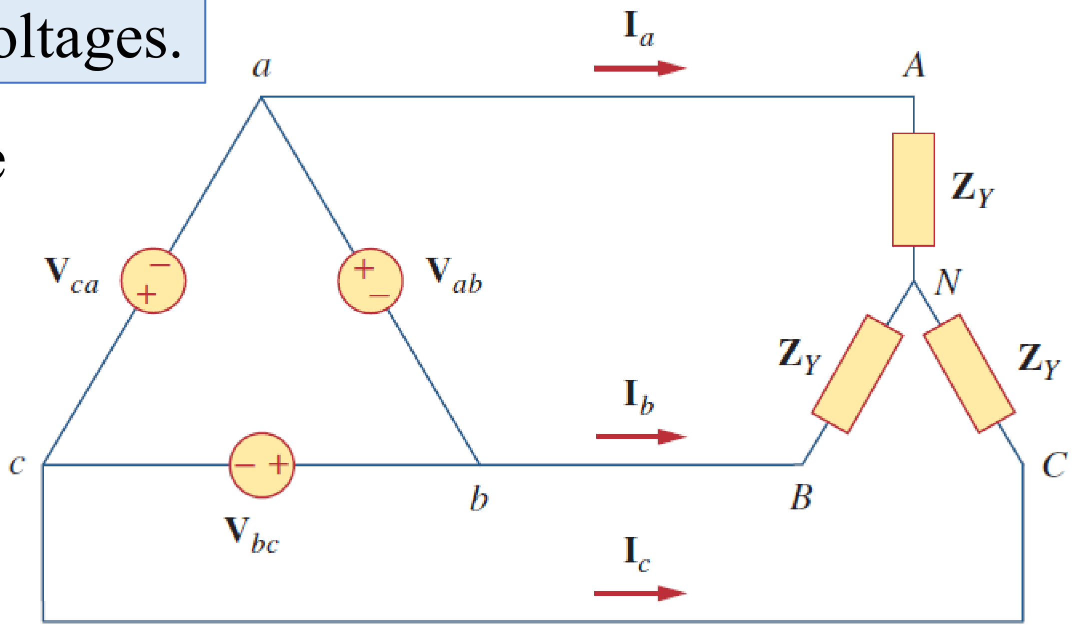


Figure 5: A balanced Δ- Y system.

Assist. Lect. Ibrahim I. Ibrahim

Another way to obtain the line currents is to replace the Δ-connected source with its equivalent Y-connected source. But, If the Δ- connected source has source impedance per phase Z_s, the equivalent Y-connected source will have a source impedance of per phase ${}^{Z_s}/_{3}$.

Alternatively, we may transform the Y-connected load to an equivalent Δ- connected load. This results in a Δ-Δ system,

As stated earlier, the Δ -connected load is more desirable than the Y-connected load. It is easier to alter the loads in any one phase of the Δ -connected loads, as the individual loads are connected directly across the lines. However, the Δ-connected source is hardly used in practice because any slight imbalance in the phase voltages will result in unwanted circulating currents.

Ex4: A balanced abc-sequence Δ-connected source with line voltage of 210 V is connected to a Y −connected balanced load $(40 + j25)\Omega$. Calculate the phase currents, use V_{ab} as reference.

$\boldsymbol{Z}_{\mathrm{Y}} = 40 + \mathrm{j}25 = 47.17\angle 32^{\circ}\boldsymbol{\Omega}$ $V_{ab} = 210\angle 0^{\circ}\mathrm{V}$

When the Δ -connected source is transformed to a Y-connected source,

$$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle{-30^{\circ}} \Longrightarrow V_{an} = 121.2\angle -30^{\circ}\ V$$

The line currents which are the same as the phase currents are:

$$\boxed{I_a = \frac{V_{an}}{Z_Y} = \frac{121.2\angle -30^{\circ}}{47.17\angle 32^{\circ}} = 2.57\angle -62^{\circ}\ A}$$

$$\boxed{I_{\mathrm{b}} = I_{\mathrm{a}}\angle{-120^{\circ}} = 2.57\angle -182^{\circ}\ A}$$

$$\boxed{I_{\mathrm{c}} = I_{\mathrm{a}}\angle{+120^{\circ}} = 2.57\angle 58^{\circ} A}$$

Summary of phase and line voltages/currents for balanced three-phase systems (abc sequence).

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

Homework

1) *Practice Problem **12.2** (page 512)*
2) *Practice Problem **12.3** (page 514)*
3) *Practice Problem **12.4** (page 516)*
4) *Practice Problem **12.5** (page 519)*

The practice problems mentioned above, can be found on the textbook of ***"Fundamentals of Electric Circuits"*** 5'th edition by Charles K. Alexander & Matthew N. O. Sadiku