University of Diyala

College of Engineering

Department of Power and Electrical Machines Engineering

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Second Year – Bachelor (BSc) – EPE 203

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EPE 203 Electric Circuits Analysis I

Lecture 3

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Lecture 3

Power in a balanced system & Unbalanced three-phase systems.

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Part 1:

Power in a balanced system

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A balanced three-phase system means:

1) All phase voltages (or currents) have equal magnitudes.
2) They are separated by $120°$ from each other in phase.
3) The loads connected to each phase are identical in impedance.

The power in a balanced three-phase system either its Y-connection or Δ-connection means the power absorbed by the load.

The power components are:

1- Active (Real) Power : P (W) (Useful work)

2- Reactive Power : Q (VAR) (Energy for magnetic fields)

3- Apparent Power : S_{app}(VA) (The total power delivered)

Phase form	Line form
$P = 3V_P I_P \cos\varphi$	$P = \sqrt{3}V_L I_L \cos\varphi$
$Q = 3V_P I_P \sin\varphi$	$Q = \sqrt{3}V_L I_L \sin\varphi$
$S_{app} = 3V_P I_P$	$S_{app} = \sqrt{3}V_L I_L$

Complex Power is a vector sum of real power and reactive power $S = P + jQ$

Apparent Power is the magnitude of the complex power, representing the total power demanded $|S| = \sqrt{P^2 + Q^2}$

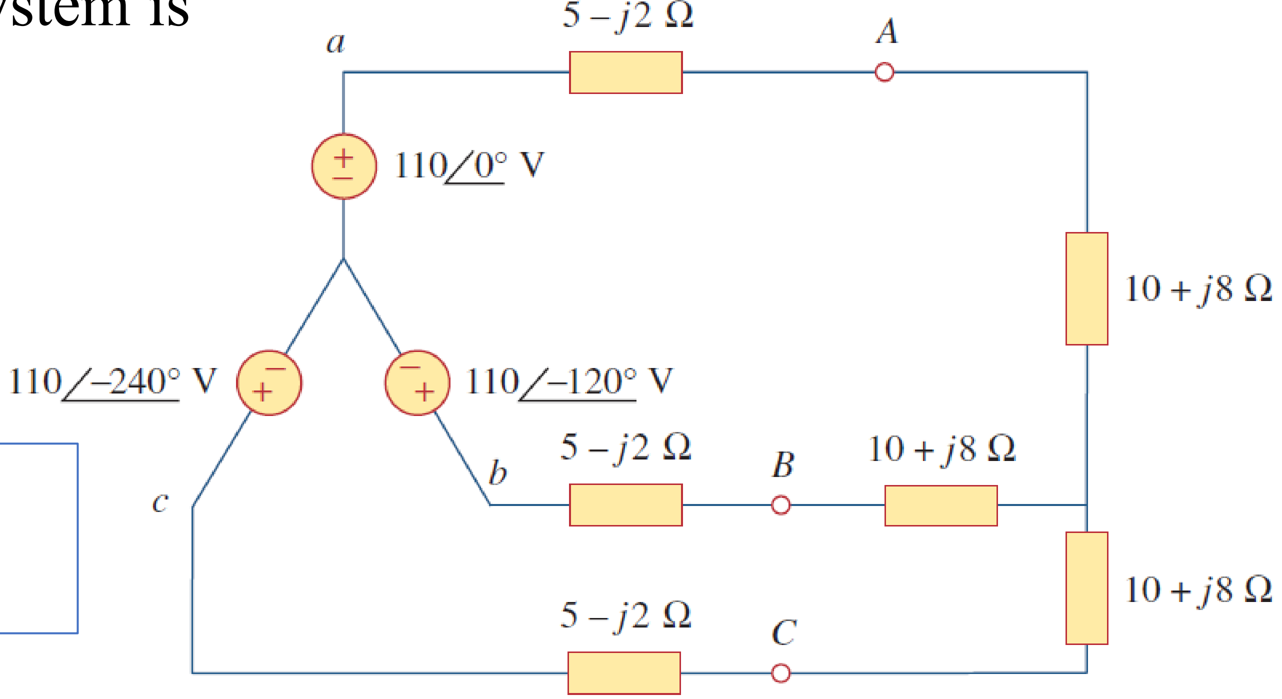
The Power Factor $\cos\varphi$ can be calculated as: $\cos\varphi = \frac{P}{S}$

φ is the phase angle between **phase voltage and phase current**

Ex1: Determine the total average power, reactive power, and complex power at the source and at the load.

$$Z_Y = Z_l + Z_L \implies Z_Y = (5 - j2) + (10 + j8) = 15 + j6\,\Omega \qquad 15 + j6\,\Omega = 16.155\angle 21.8^\circ\,\Omega$$

It is sufficient to consider one phase, as the system is balanced.



$$I_p = \frac{V_p}{Z_Y} = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ\,\text{A}$$

Thus, at the source, the complex power absorbed is

$$\mathbf{S}_s = -3\mathbf{V}_p\mathbf{I}_p^* = -3(110\underline{/0^\circ})(6.81\underline{/21.8^\circ})$$
$$= -2247\underline{/21.8^\circ} = -(2087 + j834.6)\text{ VA}$$

The real or average power absorbed is -2087 W and the reactive power is -834.6 VAR.

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2\mathbf{Z}_p$$

where $\mathbf{Z}_p = 10 + j8 = 12.81\underline{/38.66^\circ}$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81\underline{/-21.8^\circ}$. Hence,

$$\mathbf{S}_L = 3(6.81)^2 12.81\underline{/38.66^\circ} = 1782\underline{/38.66}$$
$$= (1392 + j1113)\text{ VA}$$

Ex2: A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

The apparent power is

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13 \text{ VA}$$

Since the real power is

$$P = S\cos\theta = 5600 \text{ W}$$

the power factor is

$$\text{pf} = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075$$

Part 2:

Unbalanced three-phase systems

An **unbalanced system** is due to *unbalanced voltage sources* or an *unbalanced load*.

An **unbalanced system** as shown in **Figure 1** is caused by two possible situations:

(1) The source voltages are not equal in magnitude and/or differ in phase by angles that are unequal.

(2) load impedances are unequal.

An example of an unbalanced three-phase system that consists of balanced source voltages (not shown in the **Figure 1**) and an unbalanced Y-connected load (shown in the **Figure 1**).

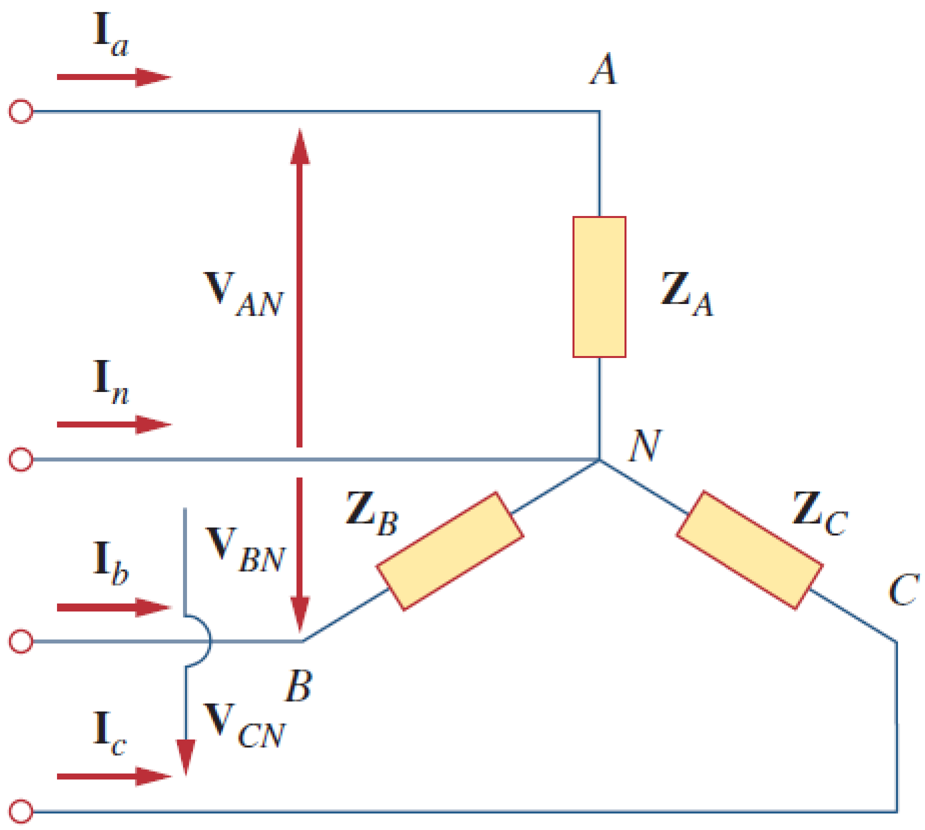


Figure 1: Unbalanced three-phase Y-connected load.

Since the load is unbalanced, Z_A, Z_B and Z_C are not equal. The line currents are determined by Ohm's law as

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C}$$

This set of **unbalanced** line currents produces current in the neutral line, which is **not zero** as in a balanced system. $\boxed{I_n = -(I_a + I_b + I_c) \neq 0}$

To calculate power in an **unbalanced** three-phase system requires that we find the power in **each phase**. The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

Ex3: The **unbalanced** Y-load of **Figure 1** has balanced voltages of 100 V and the acb sequence. Calculate the line currents and the neutral current. Take $Z_A = 15\Omega$, $Z_B = 10 + j5\ \Omega$ and $Z_C = 6 - j8\ \Omega$.

$$\mathbf{I}_a = \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ \text{ A}$$

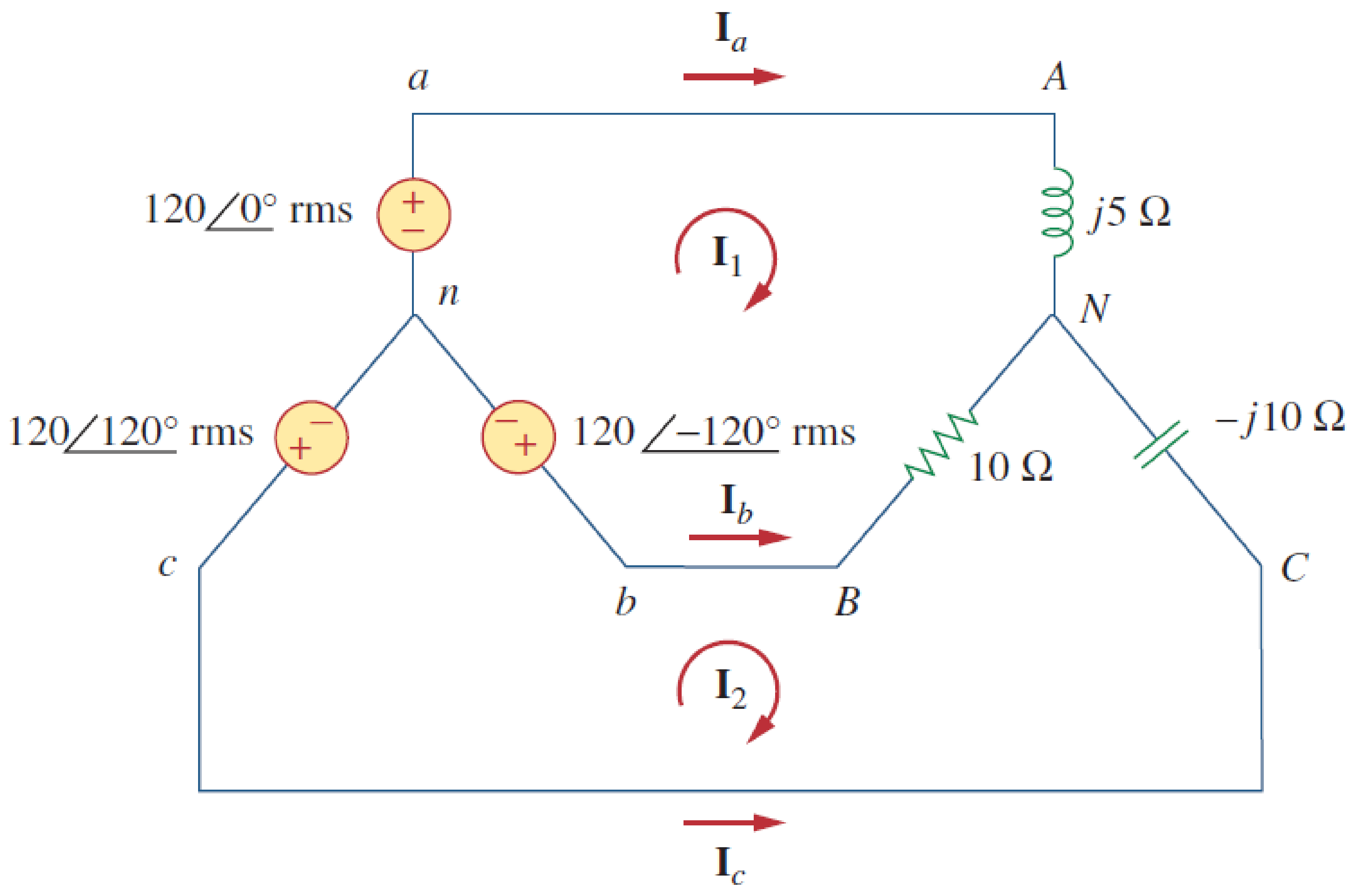
$$\mathbf{I}_b = \frac{100\angle 120^\circ}{10 + j5} = \frac{100\angle 120^\circ}{11.18\angle 26.56^\circ} = 8.94\angle 93.44^\circ \text{ A}$$

$$\mathbf{I}_c = \frac{100\angle -120^\circ}{6 - j8} = \frac{100\angle -120^\circ}{10\angle -53.13^\circ} = 10\angle -66.87^\circ \text{ A}$$

The current in the neutral line is

$$\begin{aligned}\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) &= -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)\\ &= -10.06 + j0.28 = 10.06\angle 178.4^\circ \text{ A}\end{aligned}$$

Ex4: For the unbalanced system shown below, find: (a) the line currents, (b) the total complex power absorbed by the load, and (c) the total complex power absorbed by the source.



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(a) We use mesh analysis to find the required currents.

For mesh 1, $(10 + j5)I_1 - 10I_2 = 120\sqrt{3}\angle 30^{\circ}$

For mesh 2, $-10I_1 + (10 - j10)I_2 = 120\sqrt{3}\angle -90^{\circ}$

$$\begin{bmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3}\angle 30^{\circ} \\ 120\sqrt{3}\angle -90^{\circ} \end{bmatrix}$$

Now, from the above mesh 1 and mesh 2 equations we obtained the matrix, then we have to calculate the determinants so as to find the I_1 and I_2.

The determinants are

$$\Delta = \begin{bmatrix} 10+j5 & -10 \\ -10 & 10-j10 \end{bmatrix} = 70.71\angle -45^{\circ}$$

$$\Delta_1 = \begin{bmatrix} 120\sqrt{3}\angle 30^{\circ} & -10 \\ 120\sqrt{3}\angle -90^{\circ} & 10-j10 \end{bmatrix} = 4015\angle -45^{\circ}$$

$$\Delta_2 = \begin{bmatrix} 10+j5 & 120\sqrt{3}\angle 30^{\circ} \\ -10 & 120\sqrt{3}\angle -90^{\circ} \end{bmatrix} = 3023.4\angle -20.1^{\circ}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4015\angle -45^{\circ}}{70.71\angle -45^{\circ}} = 56.78\text{ A} \qquad I_2 = \frac{\Delta_2}{\Delta} = \frac{3023.4\angle -20.1^{\circ}}{70.71\angle -45^{\circ}} = 42.75\angle 24.9^{\circ}\text{A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_1 = 56.78\text{ A}, \qquad \mathbf{I}_c = -\mathbf{I}_2 = 42.75\underline{/-155.1^{\circ}}\text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = 38.78 + j18 - 56.78 = 25.46\underline{/135^{\circ}}\text{ A}$$

(b) We can now calculate the complex power absorbed by the load.

For phase A,

$$\mathbf{S}_A = |\mathbf{I}_a|^2\mathbf{Z}_A = (56.78)^2(j5) = j16{,}120 \text{ VA}$$

For phase B,

$$\mathbf{S}_B = |\mathbf{I}_b|^2\mathbf{Z}_B = (25.46)^2(10) = 6480 \text{ VA}$$

For phase C,

$$\mathbf{S}_C = |\mathbf{I}_c|^2\mathbf{Z}_C = (42.75)^2(-j10) = -j18{,}276 \text{ VA}$$

The total complex power absorbed by the load is

$$\mathbf{S}_L = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 6480 - j2156 \text{ VA}$$

(c) We check the result above by finding the power absorbed by the source. For the voltage source in phase a,

$$\mathbf{S}_a = -\mathbf{V}_{an}\mathbf{I}_a^* = -(120\underline{/0^\circ})(56.78) = -6813.6 \text{ VA}$$

For the source in phase b,

$$\begin{aligned}\mathbf{S}_b = -\mathbf{V}_{bn}\mathbf{I}_b^* &= -(120\underline{/-120^\circ})(25.46\underline{/-135^\circ}) \\ &= -3055.2\underline{/105^\circ} = 790 - j2951.1 \text{ VA}\end{aligned}$$

For the source in phase c,

$$\begin{aligned}\mathbf{S}_c = -\mathbf{V}_{bn}\mathbf{I}_c^* &= -(120\underline{/120^\circ})(42.75\underline{/155.1^\circ}) \\ &= -5130\underline{/275.1^\circ} = -456.03 + j5109.7 \text{ VA}\end{aligned}$$

The total complex power absorbed by the three-phase source is

$$\mathbf{S}_s = \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c = -6480 + j2156 \text{ VA}$$