University of Diyala

College of Engineering

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EPE 203 Electric Circuits Analysis I

Lecture 4

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Lecture 4

The Concept of Mutual Inductance, Polarity, and the Dot Convention

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Before tackling mutual inductance, recall that self-inductance (L) is the property of a single coil where a change in current induces a voltage opposing that change:

$$v = L\frac{di}{dt}$$

where

- v is the self-induced voltage (volts),
- L is the self-inductance (henries, H),
- $\frac{di}{dt}$ is the rate of change of current.

This arises because a changing current produces a changing magnetic flux, which induces an EMF (electromotive force) opposing the change — **Lenz's Law**.

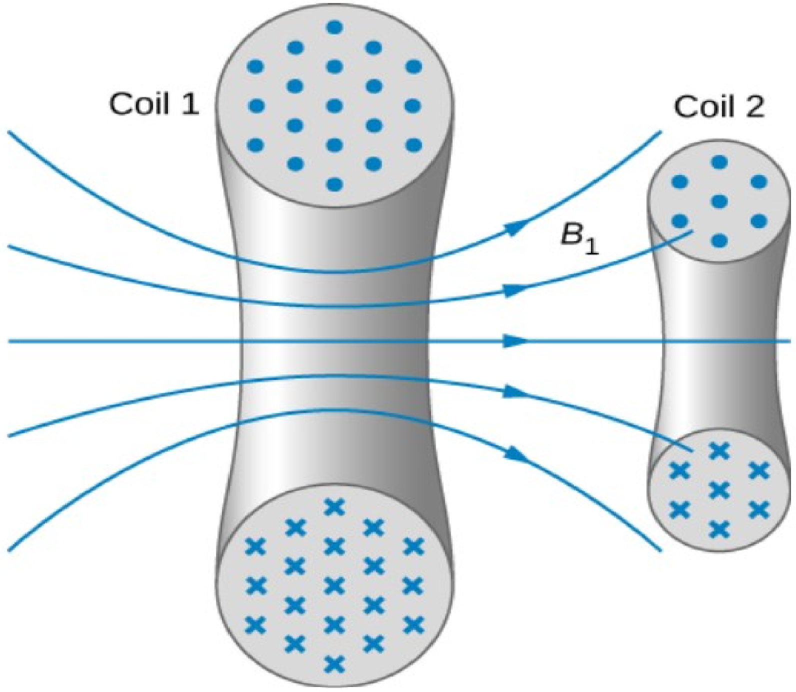
When two coils are placed so that the magnetic field produced by one links with the other, a curious phenomenon arises — the voltage induced in one coil depends not only on its own current but also on the current in its neighbor. This magnetic coupling between coils is called **mutual inductance**. This concept forms the foundation for **transformers**, **inductive sensors**, **coupled circuits**, and **wireless energy transfer** systems.

Now imagine **two coils**, coil 1 and coil 2, close enough that part of the magnetic flux created by one passes through the other.

When the current in coil 1 changes, the time-varying magnetic field it produces links with coil 2, inducing a voltage in it.

Similarly, if the current in coil 2 changes, it can induce a voltage in coil 1.

This interaction is called **mutual inductance (M)**.



Parameter	Description
N_1	Number of turns in coil 1
N_2	Number of turns in coil 2
Φ_{12}	Portion of magnetic flux linking coil 2 due to current I_1 in coil 1
Φ_{21}	Portion of magnetic flux linking coil 1 due to current I_2 in coil 2
Φ_1	The magnetic flux emanating from coil 1
Φ_2	The magnetic flux emanating from coil 2
M_{12}	The mutual inductance **as seen by coil 2** due to current in coil 1
M_{12}	The mutual inductance **as seen by coil 1** due to current in coil 2
k	Coefficient of coupling

$$\boxed{M_{21} = \frac{N_2 \Phi_{12}}{I_1}}$$

$$\boxed{\Phi_1 = \Phi_{11} + \Phi_{12}}$$

$$\boxed{M_{12} = \frac{N_1 \Phi_{21}}{I_2}}$$

$$\boxed{\Phi_2 = \Phi_{21} + \Phi_{22}}$$

$$\boxed{M = M_{21} = M_{12}}$$

$$\boxed{M = k\sqrt{L_1 L_2}}$$

$$\boxed{k = \frac{\Phi_{12}}{\Phi_1} = \frac{\Phi_{12}}{\Phi_{11} + \Phi_{12}}}$$

$$\boxed{k = \frac{\Phi_{21}}{\Phi_2} = \frac{\Phi_{21}}{\Phi_{22} + \Phi_{21}}}$$

$$\boxed{0 \le k \le 1}$$

Induced Voltages in Coupled Coils

For two magnetically coupled coils:

$$v_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

The **sign** of the mutual term depends on the **relative polarity** of the coils — that is, whether the induced voltage **aids** or **opposes** the self-induced voltage.

This is where **polarity and the dot convention** come into play.

Polarity of Induced Voltages

The polarity of the induced voltage in the secondary coil depends on **Lenz's Law**:

The induced EMF always acts in such a direction as to oppose the change in current that produced it.

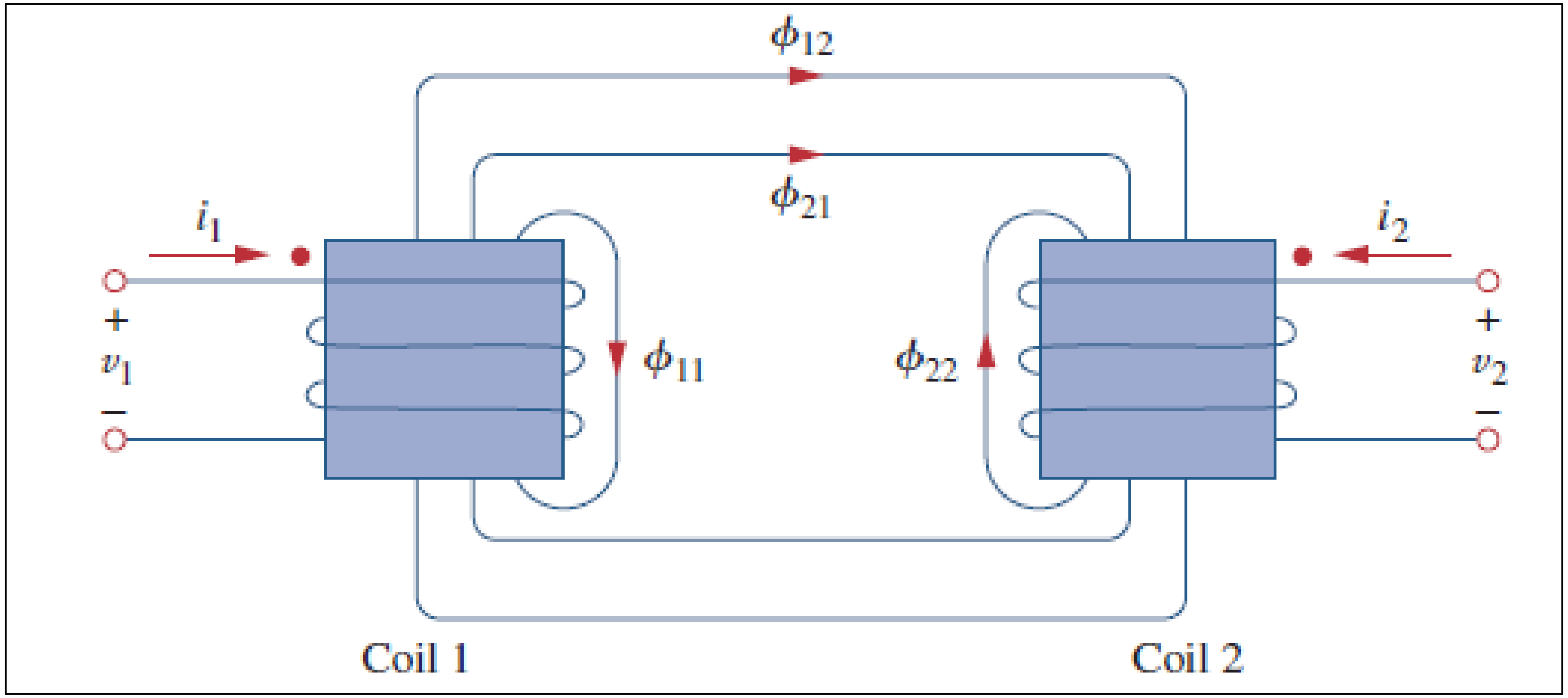
If the current entering one terminal of coil 1 produces a magnetic flux that enters the dotted terminal of coil 2, then both dotted terminals are said to have the same **polarity**.

If the flux produced by coil 1 leaves the dotted end of coil 2, they are of **opposite polarity**.

The Dot Convention

To identify relative polarity, each coil in a pair is marked with a **dot (•)** near one terminal.

The **dot convention** tells us how the voltage and current directions are related between coils.



Rules of the Dot Convention:

1) Current enters a dotted terminal of one coil → the induced voltage in the other coil has positive polarity at its dotted terminal.
2) Current leaves a dotted terminal of one coil → the induced voltage in the other coil has negative polarity at its dotted terminal.
3) The dots define whether the coils are aiding or opposing each other magnetically.

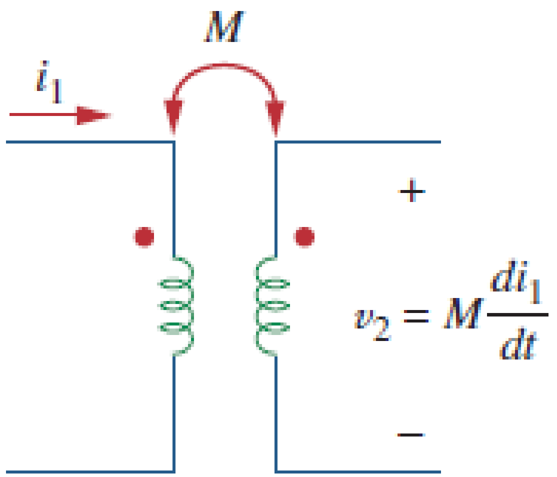
Thus, the reference polarity of the mutual voltage depends on the reference direction of the inducing current and the dots on the coupled coils.

The **four fundamental cases** of applying the **dot convention** for mutually coupled inductors are shown in the next slides in details.

Case (a)

- **Current:** i_1 is the source current in the left coil.
- **Current Entry:** i_1 **enters** the **dotted** terminal of the left coil.
- **Induced Voltage Polarity:** Following the dot convention, the voltage v_2 induced in the right coil must be **positive at its dotted terminal**.
- **Resulting Equation:** The voltage v_2 is defined as positive at the dotted terminal, which matches the required polarity.

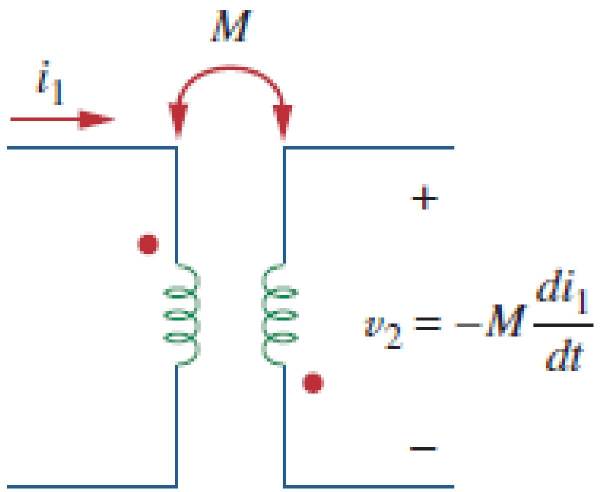
$$v_2 = M\frac{di_1}{dt}$$



Case (b)

- **Current:** i_1 is the source current in the left coil.
- **Current Entry:** i_1 **enters** the **dotted** terminal of the left coil.
- **Induced Voltage Polarity:** Following the dot convention, the voltage induced in the right coil must be **positive at its dotted terminal**.
- **Resulting Equation:** The voltage v_2 is defined as positive at the **undotted** terminal (opposite to the required polarity). Thus, a negative sign is needed in the equation.

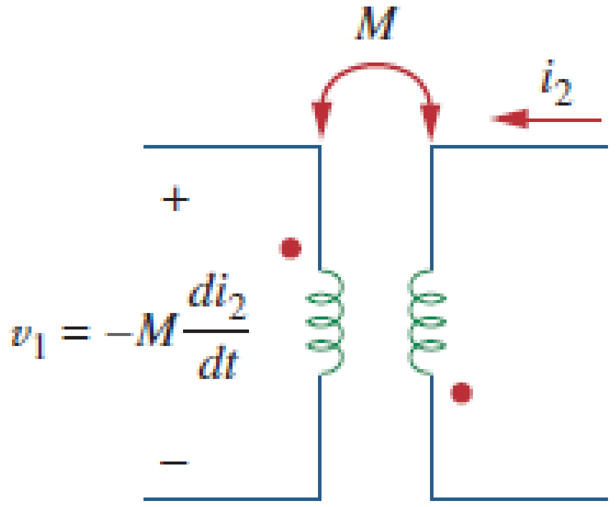
$$v_2 = -M\frac{di_1}{dt}$$



Case (c)

- **Current:** i_2 is the source current in the right coil.
- **Current Entry:** i_2 **enters** the **undotted** terminal of the right coil.
- **Induced Voltage Polarity:** The voltage v_1 induced in the left coil must be **negative at its dotted terminal** (or positive at its undotted terminal).
- **Resulting Equation:** The voltage v_1 is defined as positive at the **dotted** terminal. Since the induced voltage is positive at the *undotted* terminal, a negative sign is needed.

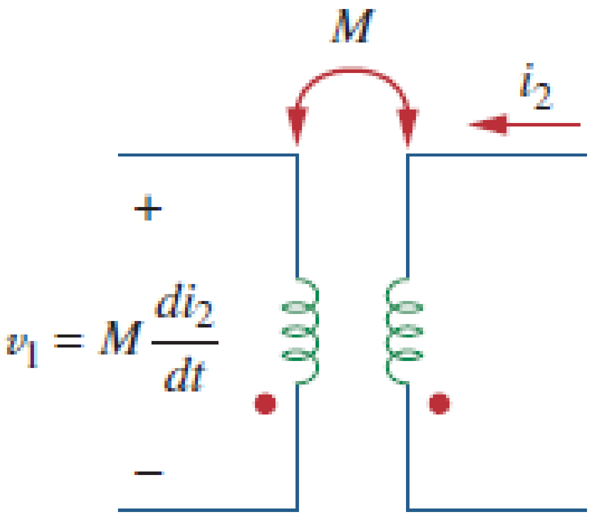
$$v_1 = -M\frac{di_2}{dt}$$



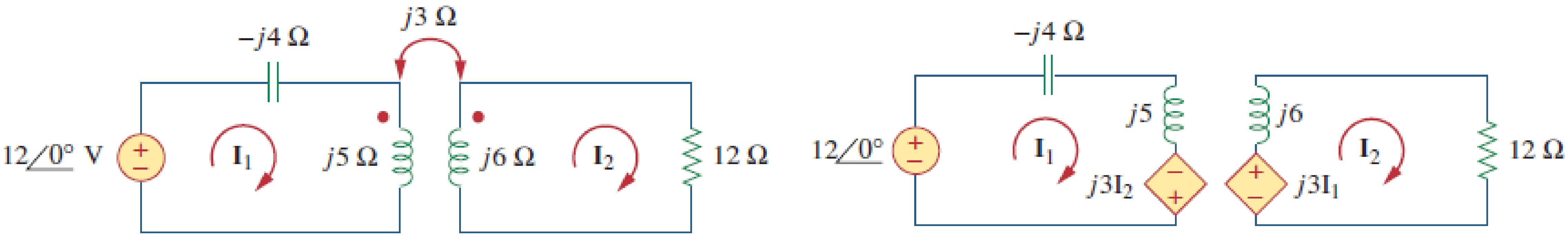
Case (d)

- **Current:** i_2 is the source current in the right coil.
- **Current Entry:** i_2 **enters** the **undotted** terminal of the right coil.
- **Induced Voltage Polarity:** The voltage v_1 induced in the left coil must be **negative at its dotted terminal** (or positive at its undotted terminal).
- **Resulting Equation:** The voltage v_1 is defined as positive at the **undotted** terminal, which matches the required induced polarity.

$$v_1 = M\frac{di_2}{dt}$$



Ex1: Calculate the phasor currents I_1 and I_2 for the circuit shown below.



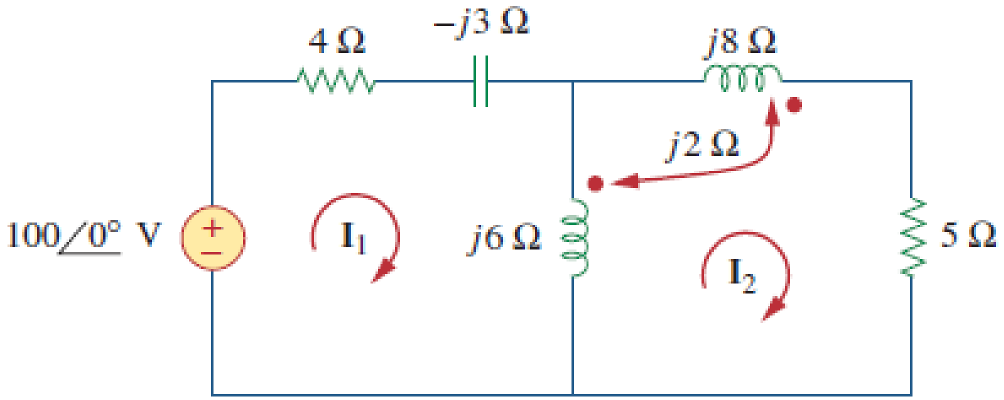
$$\begin{cases} jI_1 - j3I_2 = 12 \ \ldots\ldots\textit{(From loop 1)} \\ -j3I_1 + (12 + j6)I_2 = 0 \ \ldots\ldots\textit{(From loop 2)} \end{cases} \Rightarrow I_1 = \frac{(12 + j6)I_2}{3} = (2 - j4)I_2 \ (Eq. 1)$$

$$(j2 + 4 - j3)I_2 = 12 \Rightarrow I_2 = \frac{12}{4-j} = 2.91\angle 14.04^\circ A \ \textit{(Substituting this in Eq. 1)}$$

$$I_1 = (2 - j4)I_2 = (4.472\angle{-63.43^\circ})(2.91\angle 14.04^\circ) = 13.01\angle - 49.39^\circ A$$

Ex2: Calculate the mesh currents in the circuit shown below.

$$\begin{cases}(4+j3)I_1 - j8I_2 = 100 \text{ ……. (From loop 1)} \\ -j8I_1 + (5+j18)I_2 = 0 \text{ ……. (From loop 2)}\end{cases}$$



$$\begin{bmatrix}4+j3 & -j8 \\ -j8 & 5+j18\end{bmatrix}\begin{bmatrix}I_1 \\ I_2\end{bmatrix} = \begin{bmatrix}100 \\ 0\end{bmatrix}$$

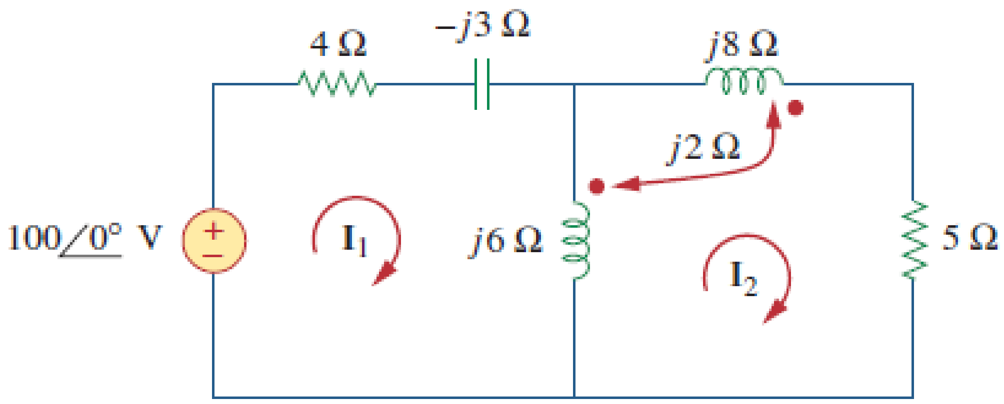
$$\Delta = \begin{bmatrix}4+j3 & -j8 \\ -j8 & 5+j18\end{bmatrix} = 30 + j87$$

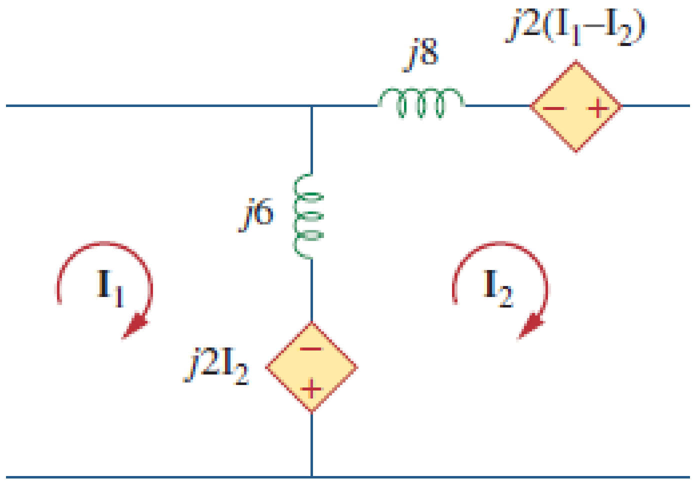
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100(5+j18)}{30+j87} = 20.3\angle 3.5^{\circ} A$$

$$\Delta_1 = \begin{bmatrix}100 & -j8 \\ 0 & 5+j18\end{bmatrix} = 100(5+j18)$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{j800}{30+j87} = 8.693\angle 19^{\circ} A$$

$$\Delta_2 = \begin{bmatrix}4+j3 & 100 \\ -j8 & 0\end{bmatrix} = j800$$


4 Ω
−j3 Ω
j8 Ω
j2 Ω
100/0° V
I1
j6 Ω
I2
5 Ω


j8
j2(I1−I2)
j6
I1
I2
j2I2

Homework

1) *Practice Problem* ***13.1*** *(page 562)*

The practice problems mentioned above, can be found on the textbook of ***"Fundamentals of Electric Circuits"*** 5'th edition by Charles K. Alexander & Matthew N. O. Sadiku