University of Diyala

College of Engineering

Department of Power and Electrical Machines Engineering

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Second Year – Bachelor (BSc) – EPE 203

Fall Semester

EPE 203 Electric Circuits Analysis I

Lecture 6

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Lecture 6

Ideal Transformers

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In the previous lecture, the analysis of coupled inductors, mutual inductance, and energy in magnetically coupled circuits led to the concept of the linear transformer. There, the focus was on how two coils share magnetic flux, how mutual inductance arises, and how coupled circuits store and transfer electromagnetic energy.

Our lecture today advances that foundation by moving from real, physical magnetic coupling to idealized model that greatly simplify circuit analysis.

This idealized version—**Ideal Transformers**—is foundational to power engineering, electronics, and control systems.

Transition from Coupled Inductors to Ideal Transformers

From Lecture 5, two coupled inductors L_1 and L_2 with mutual inductance M follow:

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

In an **ideal transformer**, the magnetic coupling is assumed to be perfect:

- All flux of coil 1 links coil 2
- All flux of coil 2 links coil 1
- Coupling coefficient $k = 1$
- No leakage flux
- No copper loss
- No core loss
- Infinite core permeability

Thus:

$$M = \sqrt{L_1 L_2}$$

This allows the model to collapse into a simple turns-ratio relationship.

Definition of the Ideal Transformer

An **ideal transformer** is a lossless, perfectly coupled device consisting of two windings:

- Primary winding with N_1 turns
- Secondary winding with N_2 turns

Assumptions:

1. No winding resistance
2. No leakage inductance
3. Infinite magnetizing inductance (zero magnetizing current)
4. Perfect magnetic coupling: $k = 1$
5. Power is conserved:

$$p_{\text{in}} = p_{\text{out}}$$

Voltage and Current Relations

Voltage Ratio

From Faraday's law:

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a$$

Where

$$a = \frac{N_1}{N_2} \quad \text{(Turns ratio)}$$

Current Ratio

From power conservation (lossless):

$$v_1 i_1 = v_2 i_2$$

Substitute voltage ratio:

$$\left(\frac{N_1}{N_2}\right) i_1 = i_2$$

Then:

$$\frac{i_2}{i_1} = \frac{N_1}{N_2} = a$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

$$v_1 = a v_2, \qquad i_1 = \frac{i_2}{a}$$

Voltage and current transform inversely.

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Impedance Reflection

If the secondary is connected to a load Z_L, the primary "sees" an equivalent impedance:

$$Z_{\text{in}} = a^2 Z_L = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

Implication:

- A small load on the secondary may appear as a large impedance on the primary if $a > 1$.
- Transformers allow impedance matching between circuits.

Equivalent Circuit of an Ideal Transformer

The equivalent circuit is extremely simple:

- No series resistances
- No leakage inductances
- No magnetizing branch
- Only the ideal turns-ratio block

Power and Energy Flow

Because the device is ideal:

$$p_1(t) = v_1 i_1 = v_2 i_2 = p_2(t)$$

This is why ideal transformers are extremely convenient in circuit analysis.

A **step-up transformer** is one whose secondary voltage is greater than its primary voltage.

$$V_2 > V_1$$

A **step-down transformer** is one whose secondary voltage is less than its primary voltage.

$$V_2 < V_1$$

Ex1: An ideal transformer is rated at ***2400/120 V***, ***9.6 kVA***, and has ***50 turns*** on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

(a) This is a step-down transformer, since $V_1 = 2{,}400$ V $> V_2 = 120$ V.

$$n = \frac{V_2}{V_1} = \frac{120}{2{,}400} = 0.05$$

(b)

$$n = \frac{N_2}{N_1} \quad \Rightarrow \quad 0.05 = \frac{50}{N_1}$$

or

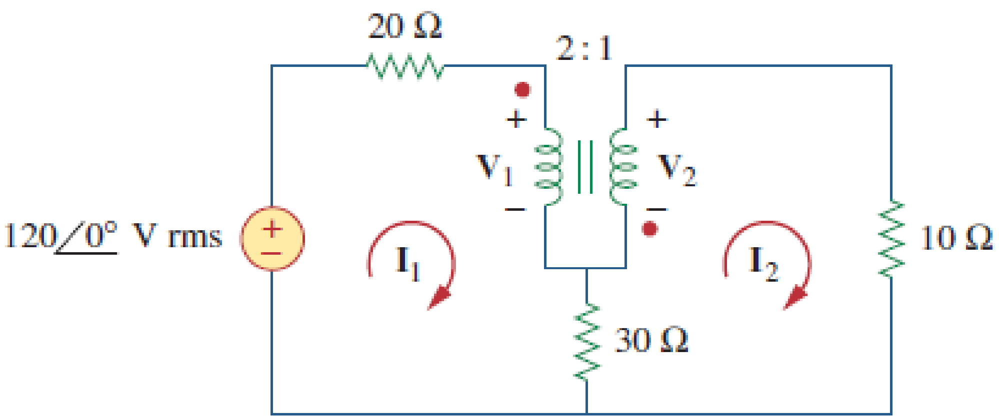
$$N_1 = \frac{50}{0.05} = 1{,}000 \text{ turns}$$

(c) $S = V_1 I_1 = V_2 I_2 = 9.6$ kVA. Hence,

$$I_1 = \frac{9{,}600}{V_1} = \frac{9{,}600}{2{,}400} = 4 \text{ A}$$

$$I_2 = \frac{9{,}600}{V_2} = \frac{9{,}600}{120} = 80 \text{ A} \qquad \text{or} \qquad I_2 = \frac{I_1}{n} = \frac{4}{0.05} = 80 \text{ A}$$

Ex2: Calculate the power supplied to the 10-Ω resistor in the ideal transformer circuit of figure below.



Reflection to the secondary or primary side cannot be done with this circuit: There is direct connection between the primary and secondary sides due to the 30- Ω resistor.

Mesh 1: $50I_1 - 30I_2 + V_1 = 120$ *..(1)*

Mesh 2: $-30I_1 + 40I_2 - V_2 = 0$ *..(2)*

At the transformer terminals,

$$\mathbf{V}_2 = -\frac{1}{2}\mathbf{V}_1 \quad ..(3)$$

$$\mathbf{I}_2 = -2\mathbf{I}_1 \quad ..(4)$$

We now have four equations and four unknowns, but our goal is to get I_2. So, we substitute for V_1 and I_1 in terms of V_2 and I_2 in Equations. (1) and (2).

$$-55I_2 - V_2 = 120$$

$$15I_2 + 40I_2 - V_2 = 0 \implies 55I_2 = V_2$$

$$-165I_2 = 120$$

$$I_2 = -\frac{120}{165} = -0.7272\,A$$

The power absorbed by the 10-Ω resistor is

$$P = (-0.7272)^2(10) = 5.3\text{ W}$$

Ex3: A load $Z_L = 25\Omega$ is connected across the secondary of an ideal transformer with $N1:N2=5:2$. What impedance does the primary "see"?

Turns ratio $a = \frac{N_1}{N_2} = \frac{5}{2} = 2.5$

Reflected impedance to primary $Z_{in} = a^2 Z_L$

$$a^2 = 2.5 * 2.5 = 6.25$$

Primary sees impedance will be equal to $Z_{in} = (6.25)(25) = 156.25\Omega$

Homework

1) *Practice Problem **13.7** (page 578)*
2) *Practice Problem **13.8** (page 579)*

The practice problems mentioned above, can be found on the textbook of ***“Fundamentals of Electric Circuits”*** 5’th edition by Charles K. Alexander & Matthew N. O. Sadiku