University of Diyala

College of Engineering

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EPE 203 Electric Circuits Analysis I

Lecture 7

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Lecture 7

Two-Port Networks

Impedance and Admittance parameters

T and Π Equivalent circuits

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Part 1
Impedance and Admittance parameters

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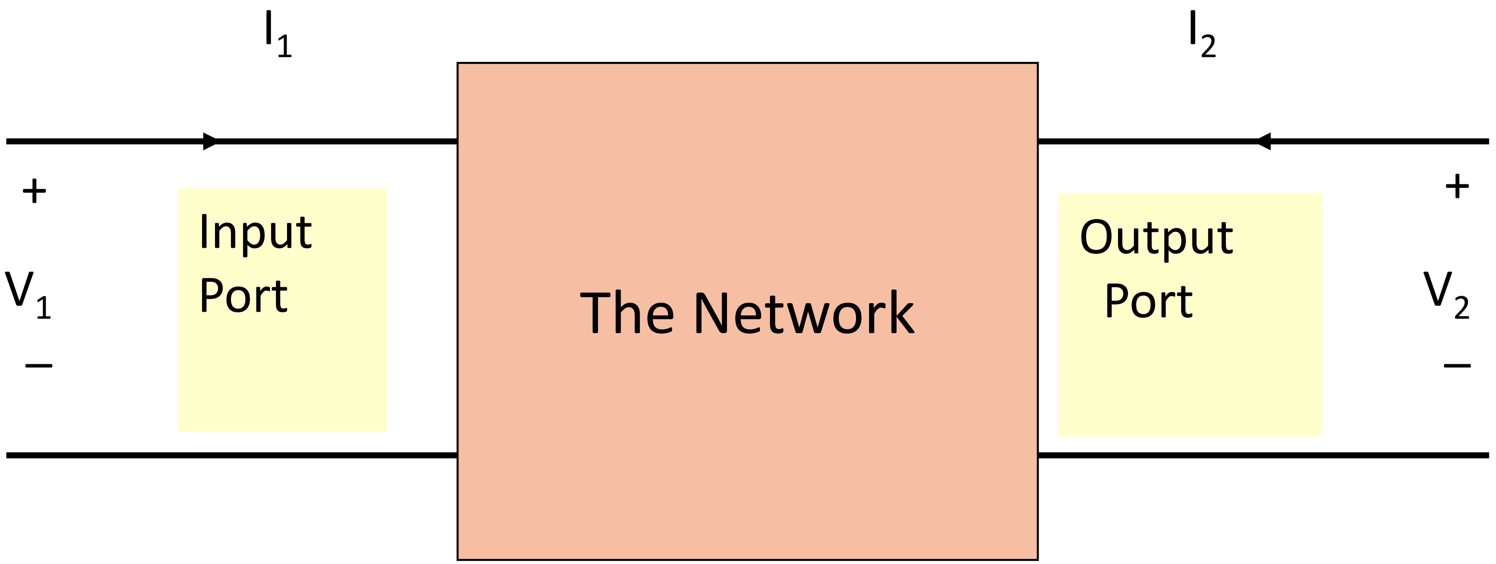
A two-port network is an electrical network with two pairs of terminals: one pair for **input (port 1)** and one pair for **output (port 2)**.

It is a useful model for analysing circuits like amplifiers, filters, transformers, and transmission lines.

The behaviour of a two-port network is described by the relationships between the voltages and currents at its terminals.

The standard configuration of a two-port network



For a general linear, time-invariant **two-port network**, the relationship between **voltages** and **currents** at the **input** and **output** terminals is described using two-port parameters. These parameters provide a systematic way to analyse circuit behaviour.

A general two-port network consists of:

Port 1: Input port with voltage V_1 and current I_1

Port 2: Output port with voltage V_2 and current I_2

The relationship between these variables is described using four key parameter sets:

1. Impedance Parameters (Z)
2. Admittance Parameters (Y)
3. Hybrid Parameters (H) (The H-parameters are a mix of Z and Y parameters)
4. Transmission (ABCD) Parameters

Z parameters:

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$\mathbf{z_{11}}$ is the impedance seen looking into port 1 when port 2 is open.

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$\mathbf{z_{12}}$ is a transfer impedance. It is the ratio of the voltage at port 1 to the current at port 2 when port 1 is open.

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$\mathbf{z_{21}}$ is a transfer impedance. It is the ratio of the voltage at port 2 to the current at port 1 when port 2 is open.

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$\mathbf{z_{22}}$ is the impedance seen looking into port 2 when port 1 is open.

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Y parameters:

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2 = 0}$$

$\mathbf{y_{11}}$ is the admittance seen looking into port 1 when port 2 is shorted.

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1 = 0}$$

$\mathbf{y_{12}}$ is a transfer admittance. It is the ratio of the current at port 1 to the voltage at port 2 when port 1 is shorted.

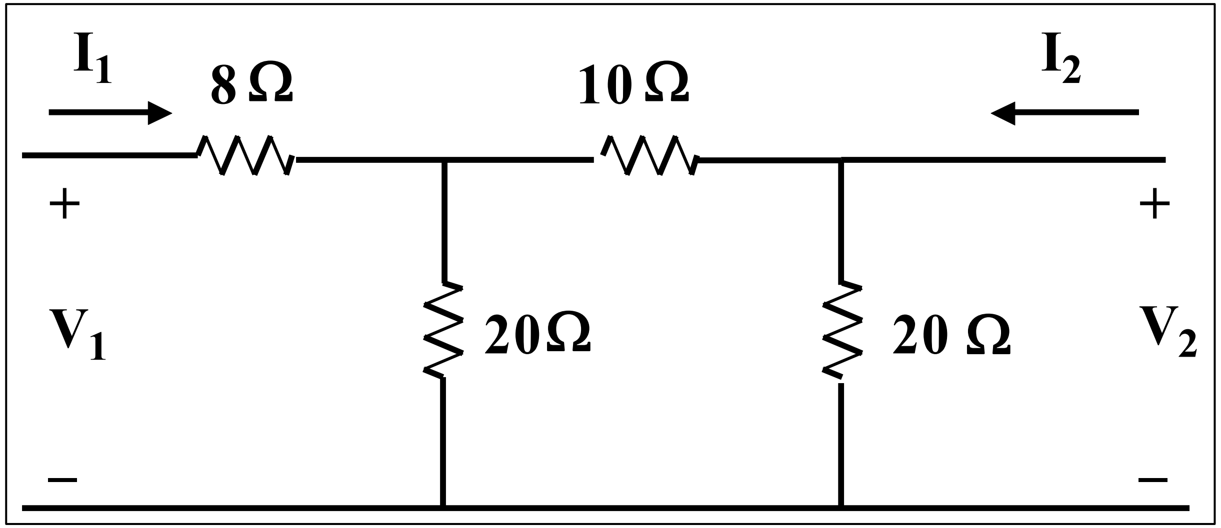
$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2 = 0}$$

$\mathbf{y_{21}}$ is a transfer impedance. It is the ratio of the current at port 2 to the voltage at port 1 when port 2 is shorted.

$$y_{22} = \frac{I_2}{V_2}\bigg|_{V_1 = 0}$$

$\mathbf{y_{22}}$ is the admittance seen looking into port 2 when port 1 is shorted.

Ex1: Given the following circuit, determine the Z parameters?



Z-Parameters relate the voltages and currents as follows:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \frac{V_1}{I_1} \Big|_{I_2 = 0}$$

$$z_{12} = \frac{V_1}{I_2} \Big|_{I_1 = 0}$$

$$z_{21} = \frac{V_2}{I_1} \Big|_{I_2 = 0}$$

$$z_{22} = \frac{V_2}{I_2} \Big|_{I_1 = 0}$$

Now the solution will be into steps:

a) Find Z_{11} and Z_{21} (Set $I_2 = 0$)

b) Find Z_{12} and Z_{22} (Set $I_1 = 0$)

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a) Find Z_{11} and Z_{21} (Set $I_2 = 0$): This means the right side of the circuit is OPEN, so the total impedance seen from port 1 can be calculated as: $Z_{11} = 8 + (20 \parallel 30) = 20\ \Omega$

The voltage V_2 at port 2 is calculated as: $V_2 = 8I_1 V$

Thus, Z_{21} is calculated as: $Z_{21} = \frac{V_2}{I_1} = \frac{8I_1}{I_1} = 8\ \Omega$

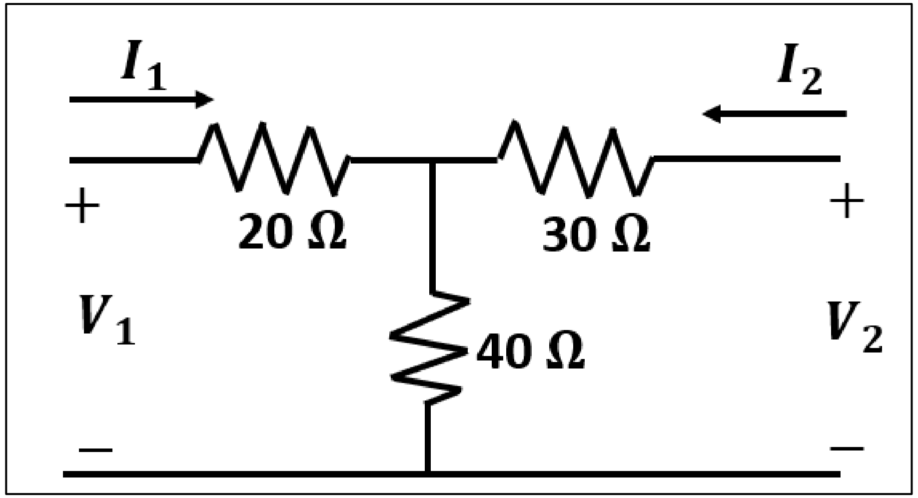
b) Find Z_{12} and Z_{22} (Set $I_1 = 0$): This means the left side of the circuit is OPEN, so the total impedance seen from port 2 can be calculated as: $Z_{22} = (20 \parallel 30) = 12\ \Omega$

The voltage V_1 at port 1 is calculated as: $V_1 = 8I_2 V$

Thus, Z_{12} is calculated as: $Z_{12} = \frac{V_1}{I_2} = \frac{8I_2}{I_2} = 8\ \Omega$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \Omega$$

Ex2: Given the following circuit, determine the Z parameters?



$$z_{11} = \frac{V_1}{I_1} = \frac{(20+40)I_1}{I_1} = 60\Omega$$

$$z_{21} = \frac{V_2}{I_1} = \frac{(40)I_1}{I_1} = 40\Omega$$

$$z_{12} = \frac{V_1}{I_2} = \frac{(40)I_2}{I_2} = 40\Omega$$

$$z_{22} = \frac{V_2}{I_2} = \frac{(30+40)I_2}{I_2} = 70\Omega$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix} \Omega$$

NOTICE

Once you have one set of parameters (Z or Y), you can convert to the other using the usual matrix inverse:

$$Y = Z^{-1} \qquad\qquad Z = Y^{-1}$$

Part 2
Equivalent Circuits for Two-Port Networks

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1. T-Equivalent Circuit

The **T-equivalent circuit** is a three-impedance model that represents a two-port network. It consists of:

1. A series impedance Z_A at the input port.
2. A series impedance Z_B at the output port.
3. A shunt impedance Z_C connecting the two series impedances.

Relationship with Z-Parameters:

The T-equivalent circuit is directly related to the **Z-parameters** of the two-port network:

$$Z_A = Z_{11} - Z_{12}$$

$$Z_B = Z_{22} - Z_{12}$$

$$Z_C = Z_{12}$$

Example:

Given the Z-parameters of a two-port network:

$$[Z] = \begin{bmatrix} 10 & 4 \\ 4 & 8 \end{bmatrix} \Omega$$

Find the T-equivalent circuit.

Solution:

$$Z_A = Z_{11} - Z_{12} = 10 - 4 = 6\,\Omega$$

$$Z_B = Z_{22} - Z_{12} = 8 - 4 = 4\,\Omega$$

$$Z_C = Z_{12} = 4\,\Omega$$

The T-equivalent circuit has:

- $Z_A = 6\,\Omega$
- $Z_B = 4\,\Omega$
- $Z_C = 4\,\Omega$

2. Π-Equivalent Circuit

The **Π-equivalent circuit** is a three-admittance model that represents a two-port network. It consists of:

1. A shunt admittance Y_A at the input port.
2. A shunt admittance Y_B at the output port.
3. A series admittance Y_C connecting the two shunt admittances.

Relationship with Y-Parameters:

The Π-equivalent circuit is directly related to the **Y-parameters** of the two-port network:

$$Y_A = Y_{11} + Y_{12}$$

$$Y_B = Y_{22} + Y_{12}$$

$$Y_C = -Y_{12}$$

Example:

Given the Y-parameters of a two-port network:

$$[Y] = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.3 \end{bmatrix} \text{ S}$$

Find the Π-equivalent circuit.

Solution:

$$Y_A = Y_{11} + Y_{12} = 0.2 + (-0.1) = 0.1\,\text{S}$$

$$Y_B = Y_{22} + Y_{12} = 0.3 + (-0.1) = 0.2\,\text{S}$$

$$Y_C = -Y_{12} = -(-0.1) = 0.1\,\text{S}$$

The Π-equivalent circuit has:

- $Y_A = 0.1\,\text{S}$
- $Y_B = 0.2\,\text{S}$
- $Y_C = 0.1\,\text{S}$

3. Conversion Between T and Π Circuits

The T and Π circuits are **dual** to each other. They can be converted using the following transformations:

T to Π Conversion:

$$Y_A = \frac{Z_B + Z_C}{Z_A Z_B + Z_B Z_C + Z_C Z_A}$$

$$Y_B = \frac{Z_A + Z_C}{Z_A Z_B + Z_B Z_C + Z_C Z_A}$$

$$Y_C = \frac{Z_A + Z_B}{Z_A Z_B + Z_B Z_C + Z_C Z_A}$$

Π to T Conversion:

$$Z_A = \frac{Y_B + Y_C}{Y_A Y_B + Y_B Y_C + Y_C Y_A}$$

$$Z_B = \frac{Y_A + Y_C}{Y_A Y_B + Y_B Y_C + Y_C Y_A}$$

$$Z_C = \frac{Y_A + Y_B}{Y_A Y_B + Y_B Y_C + Y_C Y_A}$$