University of Diyala

College of Engineering

Department of Power and Electrical Machines Engineering

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Second Year – Bachelor (BSc) – EPE 203

Fall Semester

EPE 203 Electric Circuits Analysis I

Lecture 8

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Lecture 8

Two-Port Networks

Hybrid, and Transmission parameters.

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Part 1
Hybrid parameters

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Hybrid parameters are particularly useful for modeling **transistor amplifiers**, where one equation resembles an impedance relation and the other resembles a current gain relation. The name “hybrid” refers to mixing voltage and current variables.

Hybrid parameters relate port variables as:

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Here:

- V_1 and I_2 are the **dependent** variables.
- I_1 and V_2 are the **independent** variables.

Currents and voltages in a two-port have to satisfy circuit laws, but we still get to choose which ones we treat as inputs (independent) and which ones we treat as outputs (dependent).

Hybrid parameters were invented for transistor modeling, and transistors behave in a wonderfully lopsided way. One port looks mostly like a current-driven device, and the other port looks mostly like a voltage-driven device.

That's why V_1 and I_2 are chosen as the dependent variables: the equations are solved *for* them. Nothing mystical is happening; it's a deliberate choice to match real devices.

A transistor's small-signal model gives you something like:

- input voltage depends on input current plus a little feedback from output voltage
- output current depends on input current and output voltage

h parameters:

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2 = 0}$$

Input impedance (Open-Circuit)

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1 = 0}$$

Reverse Voltage Gain (Open-Circuit)

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2 = 0}$$

Forward Current Gain (Short-Circuit)

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1 = 0}$$

Output Admittance (Short-Circuit)

Determine h_{11} and h_{21}

Set output port voltage $V_2 = 0$ (short circuit).

- Apply a known current I_1.
- Measure V_1 and I_2.

$$h_{11} = \frac{V_1}{I_1}, \quad h_{21} = \frac{I_2}{I_1}$$

Determine h_{12} and h_{22}

Set $I_1 = 0$ (open circuit at input).

- Apply a known V_2.
- Measure resulting V_1 and I_2.

$$h_{12} = \frac{V_1}{V_2}, \quad h_{22} = \frac{I_2}{V_2}$$

Part 2
Transmission parameters

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Transmission (or chain) parameters are ideal for cascaded networks. The output variables V_2, I_2 influence the input side through a simple matrix product:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

This makes ABCD parameters essential in:

- Communication lines
- Two-port filter sections
- Microwave components
- Control system modular modeling

$ABCD$ parameters:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}$$

Voltage ratio with output port open

$$B = \left.\frac{V_1}{I_2}\right|_{V_2 = 0}$$

Transfer impedance with output port short-circuited

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}$$

Transfer admittance

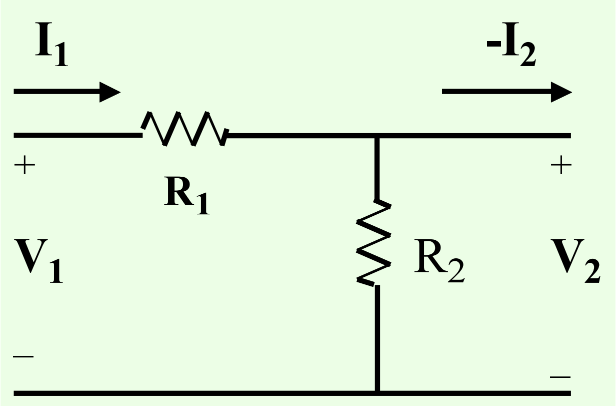
$$D = \left.\frac{I_1}{I_2}\right|_{V_2 = 0}$$

Current ratio with output short-circuited

Transmission parameters (A,B,C,D):

Example

Given the network below with assumed voltage polarities and Current directions compatible with the A,B,C,D parameters.



$V_1 = (R_1 + R_2)I_1 + R_2I_2$

$V_2 = R_2I_1 + R_2I_2$

To find the **ABCD-parameters** (also known as transmission parameters) for the given network, we need to express the input voltage V_1 and input current I_1 in terms of the output voltage V_2 and output current I_2. The ABCD-parameters are defined by the following equations:

$$\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases}$$

Step 1: Analyze the Network

The given network consists of two resistors R_1 and R_2 connected in series. The input port is at V_1 and I_1, and the output port is at V_2 and I_2. The current I_1 flows through R_1 and R_2, and the voltage V_2 is measured across R_2.

Step 2: Find A and C (Open-Circuit Conditions)

To find A and C, we set $I_2 = 0$ (open-circuit the output port).

1. **Voltage Relationship:**
   - The voltage across R_2 is V_2.
   - The voltage across R_1 is $I_1 R_1$.
   - By KVL, $V_1 = I_1 R_1 + V_2$.
   - Since $I_2 = 0$, $I_1 = \frac{V_2}{R_2}$.
   - Substituting I_1 into the KVL equation:

$$V_1 = \left(\frac{V_2}{R_2}\right) R_1 + V_2 = V_2 \left(1 + \frac{R_1}{R_2}\right)$$

   - Thus, $A = \frac{V_1}{V_2} = 1 + \frac{R_1}{R_2}$.
2. **Current Relationship:**
   - From $I_1 = \frac{V_2}{R_2}$, we have:

$$I_1 = \frac{V_2}{R_2}$$

   - Thus, $C = \frac{I_1}{V_2} = \frac{1}{R_2}$.

Step 3: Find B and D (Short-Circuit Conditions)

To find B and D, we set $V_2 = 0$ (short-circuit the output port).

1. **Voltage Relationship:**
   - With $V_2 = 0$, the voltage across R_2 is 0.
   - By KVL, $V_1 = I_1 R_1$.
   - The current I_2 is the same as I_1 (since $V_2 = 0$).
   - Thus, $V_1 = I_2 R_1$.
   - Therefore, $B = -\frac{V_1}{I_2} = R_1$.
2. **Current Relationship:**
   - Since $I_1 = I_2$, we have:

$$I_1 = I_2$$

   - Thus, $D = -\frac{I_1}{I_2} = 1$.

Step 4: Write the ABCD-Parameter Matrix

The ABCD-parameters for the given network are:

$$A = 1 + \frac{R_1}{R_2}, \quad B = R_1, \quad C = \frac{1}{R_2}, \quad D = 1$$

The ABCD-parameter matrix is:

$$[ABCD] = \begin{bmatrix} 1 + \frac{R_1}{R_2} & R_1 \\ \frac{1}{R_2} & 1 \end{bmatrix}$$

Final Answer:

The ABCD-parameters for the given network are:

$$A = 1 + \frac{R_1}{R_2}, \quad B = R_1, \quad C = \frac{1}{R_2}, \quad D = 1$$

The ABCD-parameter matrix is:

$$[ABCD] = \begin{bmatrix} 1 + \frac{R_1}{R_2} & R_1 \\ \frac{1}{R_2} & 1 \end{bmatrix}$$