University of Diyala

College of Engineering

Department of Power and Electrical Machines Engineering

Academic Year 2025-2026

Second Year – Bachelor (BSc) – EPE 204

Spring Semester

EPE 204 Electric Circuits Analysis II

Lecture 1

Assist. Lect. Ibrahim I. Ibrahim

Lecture 1

Resonance and Frequency-Selective Circuits

Assist. Lect. Ibrahim I. Ibrahim

Part 1

Series resonance and Parallel resonance

Assist. Lect. Ibrahim I. Ibrahim

Introduction to Resonance in AC Circuits

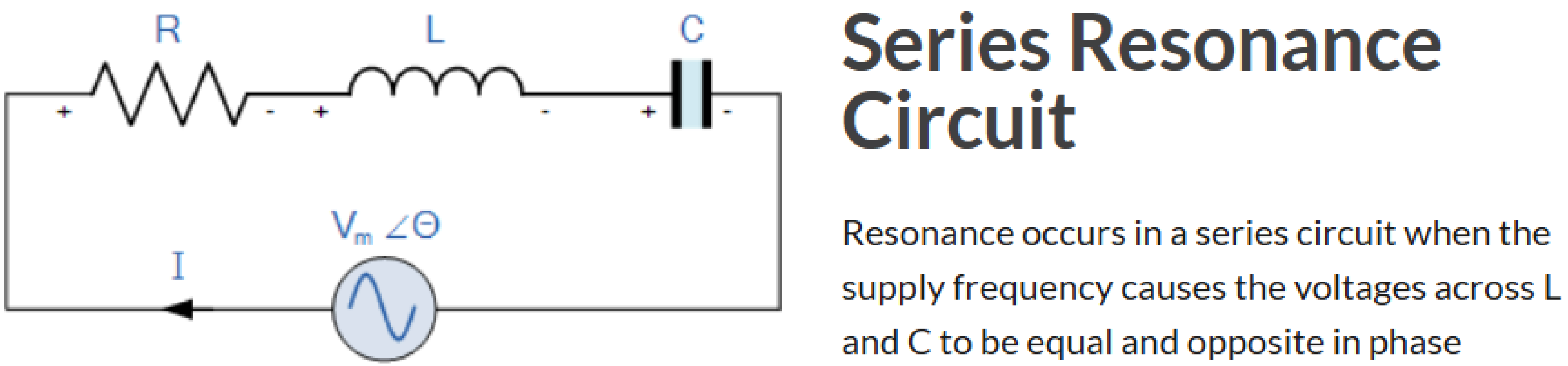
Resonance occurs in AC circuits when the inductive reactance $X_L = \omega L$ and capacitive reactance $X_C = \frac{1}{\omega C}$ are equal in magnitude but opposite in sign. This leads to maximum voltage or current, depending on the configuration.

Resonance can be classified into:

1. **Series RLC Circuits (Series Resonance)**
2. **Parallel RLC Circuits (Parallel Resonance)**

When resonance happens, the circuit behaves like it's purely resistive and the resonant frequency (f_r) is the frequency where this occurs.

Series Resonance



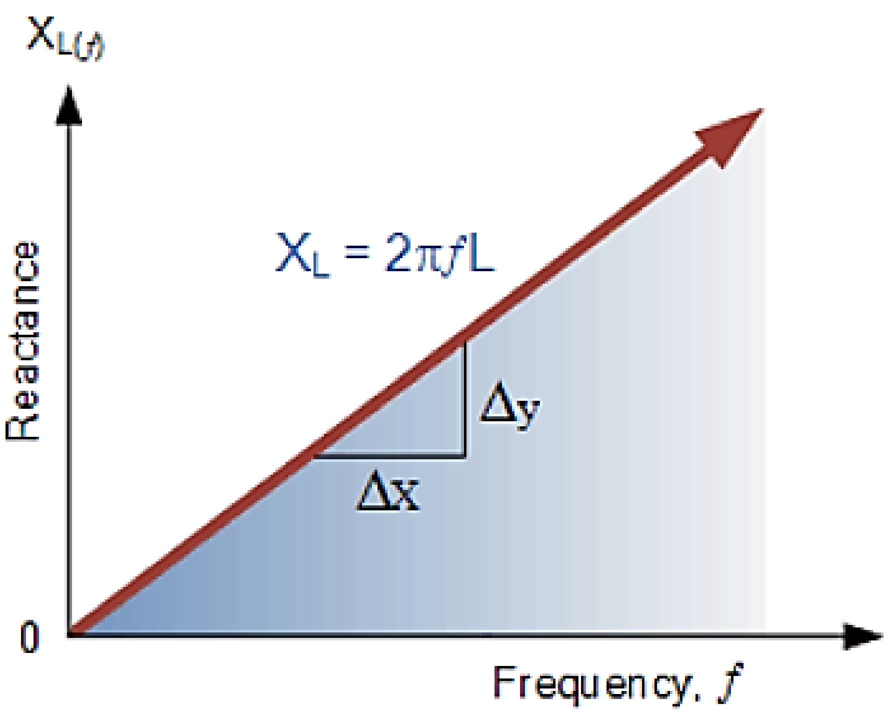
Series Resonance circuits are one of the most important circuits used electrical and electronic circuits. They can be found in various forms such as in AC mains filters, noise filters and in radio and television tuning circuits producing a very selective tuning circuit for the receiving of the different frequency channels. Consider the simple series RLC circuit below.

Series Resonance

Firstly, let us define what we already know about series RLC circuits.

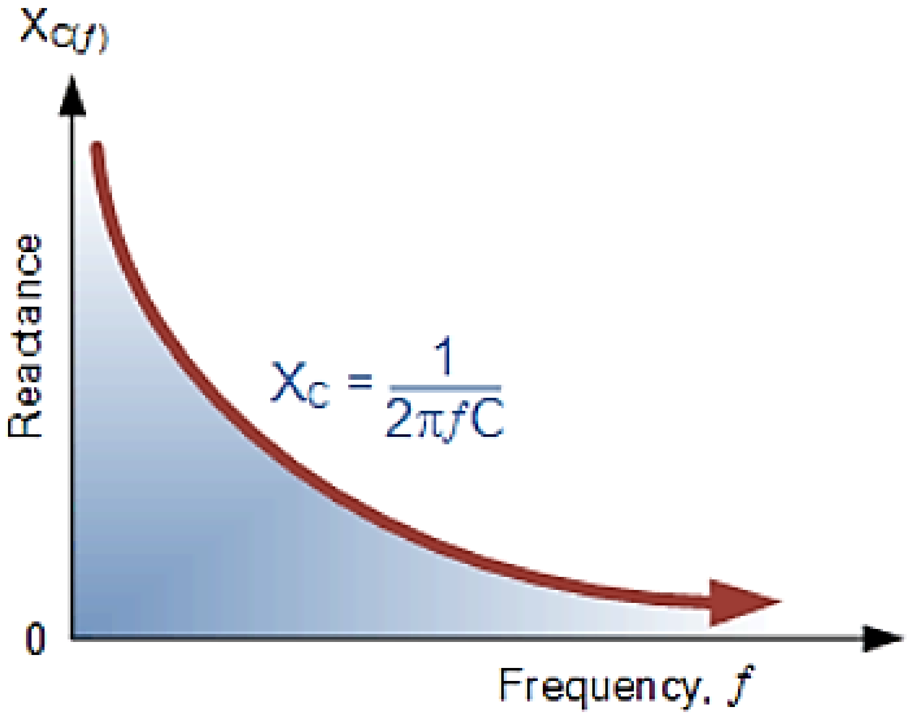
- Inductive reactance: $X_L = 2\pi f L = \omega L$
- Capacitive reactance: $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$
- When $X_L > X_C$ the circuit is Inductive
- When $X_C > X_L$ the circuit is Capacitive
- Total circuit reactance = $X_T = X_L - X_C$ or $X_C - X_L$
- Total circuit impedance = $Z = \sqrt{R^2 + X_T^2} = R + jX$

Series Resonance



X_L vs frequency

$$X_L \propto f$$



X_C vs frequency

$$X_C \propto \frac{1}{f}$$

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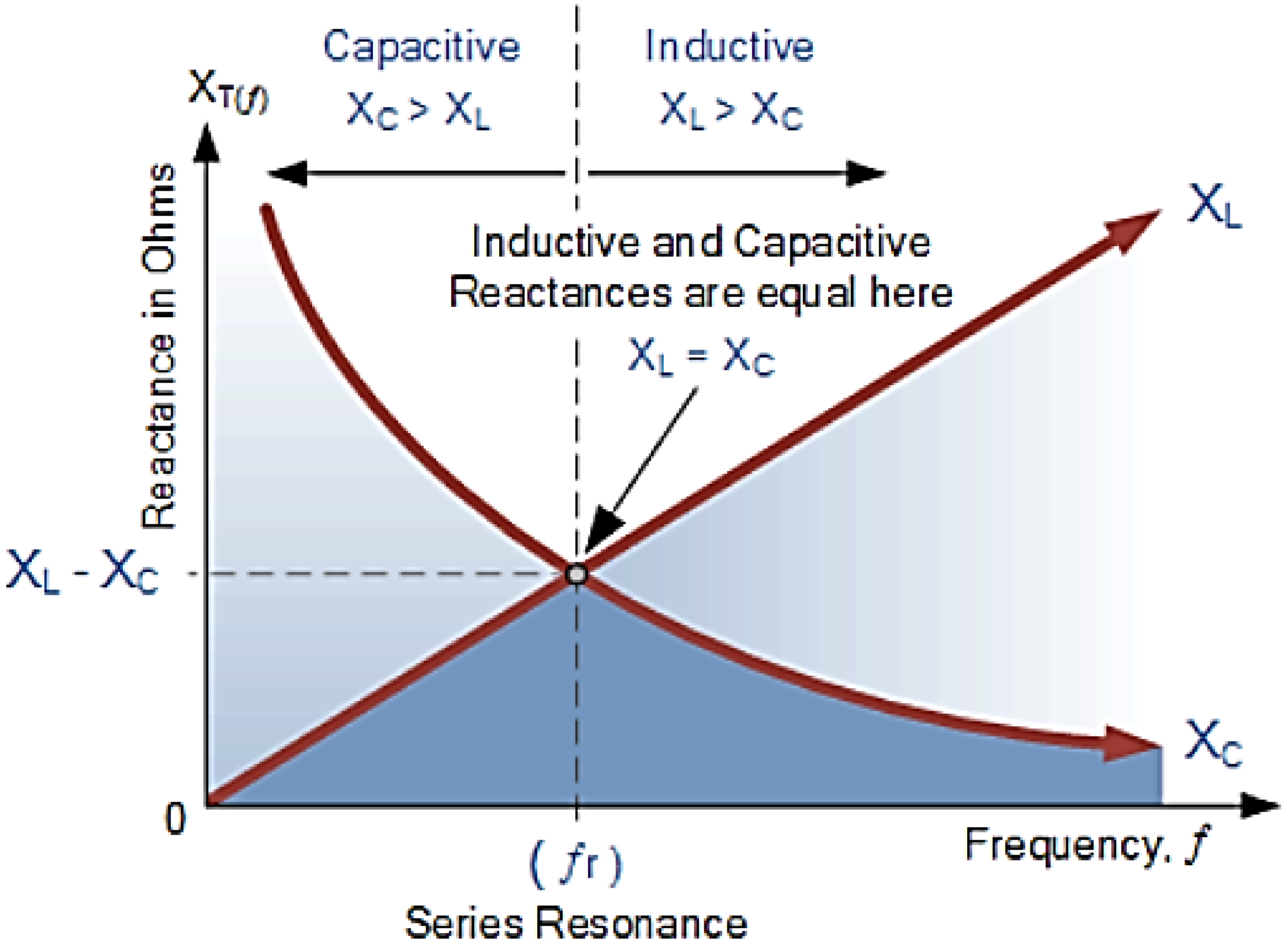
Series Resonance



Series Resonance

In a **series resonant circuit**, the **resonant frequency, f_r** point can be calculated as follows.

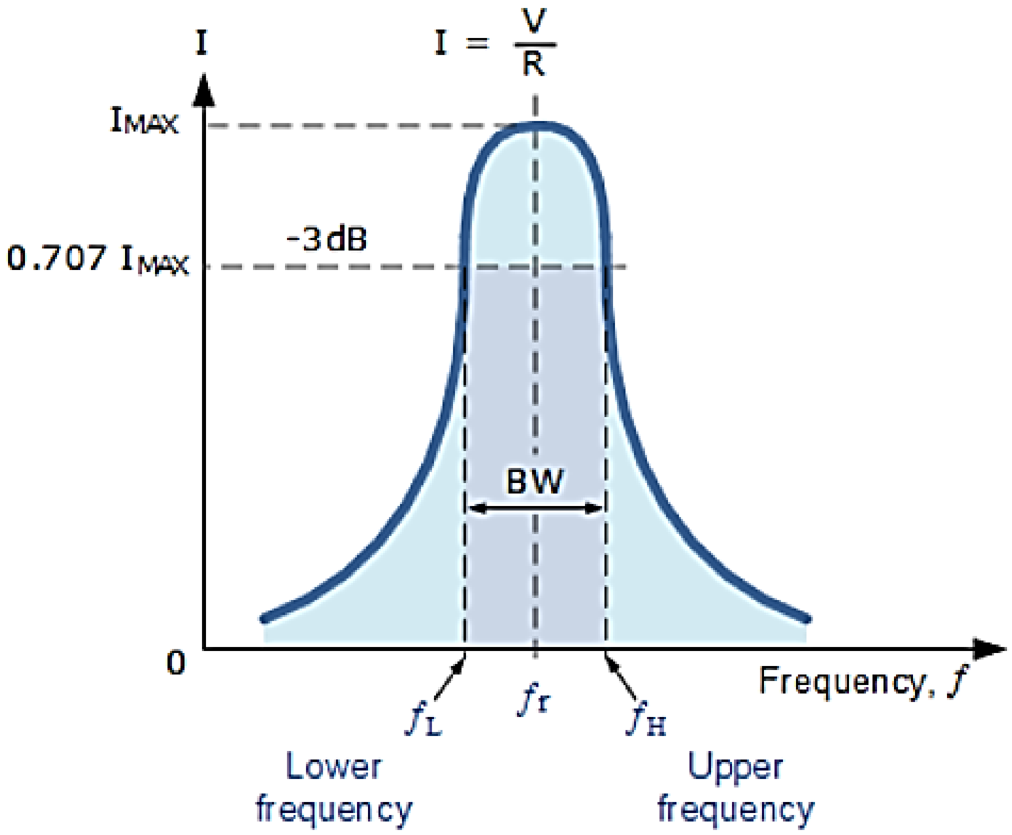
$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} \,(\text{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \,(\text{rads})$$

Bandwidth of a Series Resonance Circuit



Series Resonance

1). Resonant Frequency, (f_r)

$$X_L = X_C \quad \Rightarrow \quad \omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r^2 = \frac{1}{LC} \quad \therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

2). Current, (I)

at ω_r $\quad Z_T = \text{min}, \quad I_S = \text{max}$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{V_{max}}{\sqrt{R^2 + \left(\omega_r L - \frac{1}{\omega_r C}\right)^2}}$$

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Series Resonance

3). Lower cut-off frequency, (f_L)

$$\text{At half power, } \frac{P_m}{2}, \quad I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$Z = \sqrt{2}R, \quad X = -R \text{ (capacitive)}$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

4). Upper cut-off frequency, (f_H)

$$\text{At half power, } \frac{P_m}{2}, \quad I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$Z = \sqrt{2}R, \quad X = +R \text{ (inductive)}$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Series Resonance

5). Bandwidth, (BW)

$$BW = \frac{f_r}{Q}, \quad f_H - f_L, \quad \frac{R}{L}\ (\text{rads}) \quad \text{or} \quad \frac{R}{2\pi L}\ (\text{Hz})$$

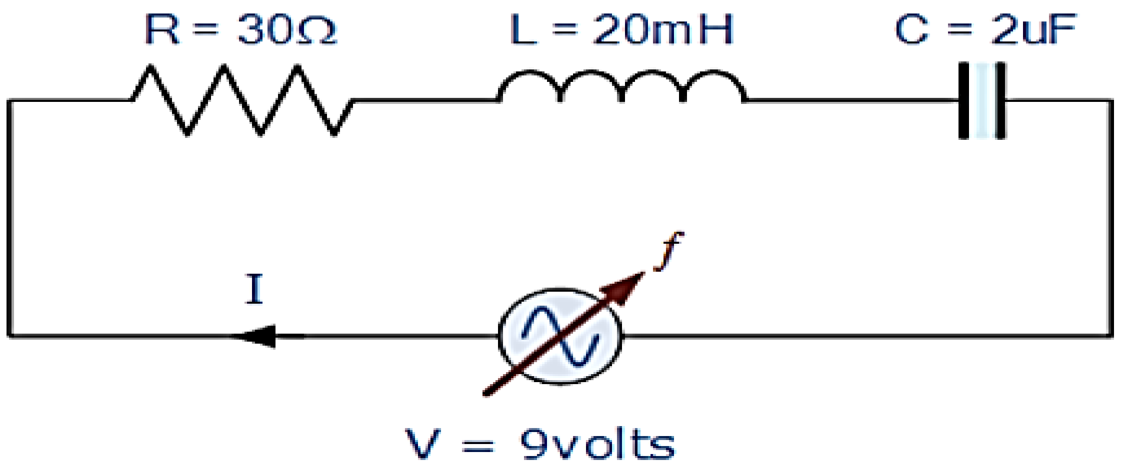
6). Quality Factor, (Q)

$$Q = \frac{\omega_r L}{R} = \frac{X_L}{R} = \frac{1}{\omega_r C R} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Series Resonance Example No1

A series resonance network consisting of a resistor of 30Ω, a capacitor of 2uF and an inductor of 20mH is connected across a sinusoidal supply voltage which has a constant output of 9 volts at all frequencies.

Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies.



Series Resonance

1. Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times 2\times 10^{-6}}} = 796\text{Hz}$$

2. Circuit Current at Resonance, I_m

$$I = \frac{V}{R} = \frac{9}{30} = 0.3\text{A} \;\text{ or }\; 300\text{mA}$$

3. Inductive Reactance at Resonance, X_L

$$X_L = 2\pi f L = 2\pi\times 796\times 0.02 = 100\Omega$$

4. Voltages across the inductor and the capacitor, V_L, V_C

$$V_L = V_C$$
$$V_L = I\times X_L = 300\text{mA}\times 100\Omega$$
$$V_L = 30\text{volts}$$

Note: the supply voltage may be only 9 volts, but at resonance, the reactive voltages across the capacitor, V_C and the inductor, V_L are 30 volts peak!

Series Resonance

5. Quality factor, Q

$$Q = \frac{X_L}{R} = \frac{100}{30} = 3.33$$

6. Bandwidth, BW

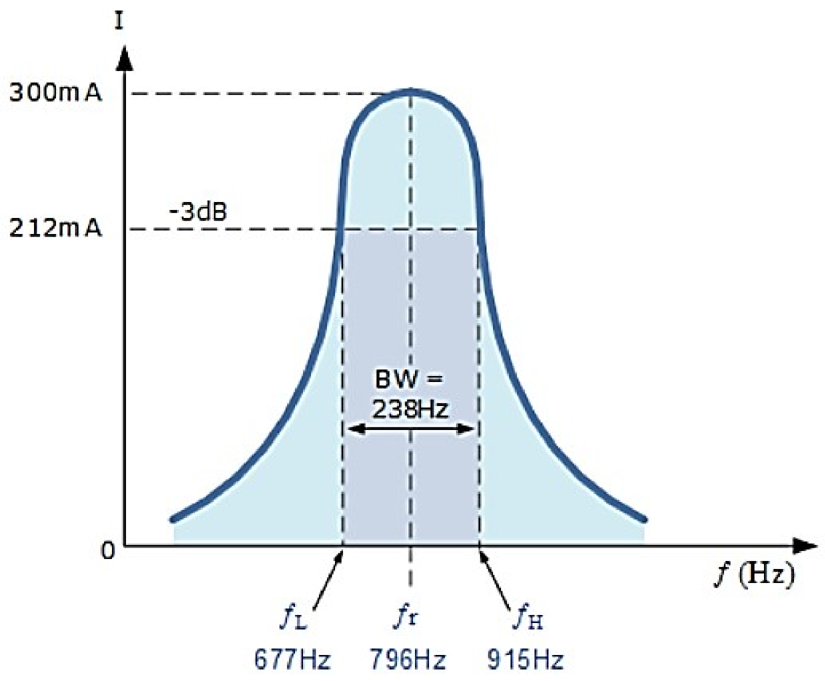
$$BW = \frac{f_r}{Q} = \frac{796}{3.33} = 238\text{Hz}$$

7. The upper and lower -3dB frequency points, f_H and f_L

$$f_L = f_r - \frac{1}{2}BW = 796 - \frac{1}{2}(238) = 677\text{Hz}$$

$$f_H = f_r + \frac{1}{2}BW = 796 + \frac{1}{2}(238) = 915\text{Hz}$$

8. Current Waveform



Series Resonance Example No2

A series circuit consists of a resistance of 4Ω, an inductance of 500mH and a variable capacitance connected across a 100V, 50Hz supply. Calculate the capacitance require to produce a series resonance condition, and the voltages generated across both the inductor and the capacitor at the point of resonance.

Resonant Frequency, f_r

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157.1\Omega$$

$$\text{at resonance: } X_C = X_L = 157.1\Omega$$

$$\therefore C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi.50.157.1} = 20.3\mu F$$

Voltages across the inductor and the capacitor, V_L, V_C

$$I_S = \frac{V}{R} = \frac{100}{4} = 25\text{Amps}$$

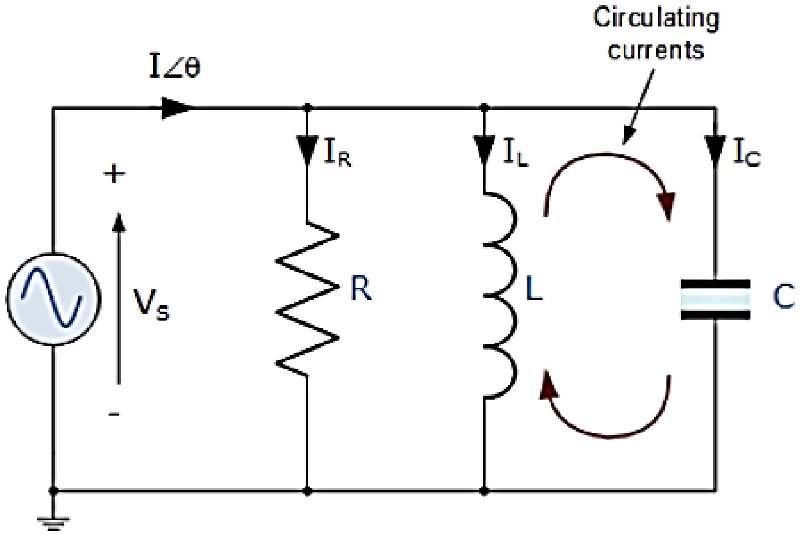
$$\text{at Resonance: } V_L = V_C$$

$$V_L = I \times X_L = 25 \times 157.1$$

$$\text{Thus } V_L = 3{,}927.5\text{volts or } 3.9\text{kV}$$

$$\text{and } V_C = 3{,}927.5\text{volts or } 3.9\text{kV}$$

Parallel Resonance



Parallel Resonance Circuit

Parallel resonance occurs when the supply frequency creates zero phase difference between the supply voltage and current producing a resistive circuit

A ***parallel resonant*** circuit stores the circuit energy in the magnetic field of the inductor and the electric field of the capacitor. This energy is constantly being transferred back and forth between the inductor and the capacitor which results in zero current and energy being drawn from the supply.

Parallel Resonance

Let us define what we already know about parallel RLC circuits.

Admittance, $Y = \frac{1}{Z} = \sqrt{G^2 + B^2}$

Conductance, $G = \frac{1}{R}$

Inductive Susceptance, $B_L = \frac{1}{2\pi f L}$

Capacitive Susceptance, $B_C = 2\pi f C$

Parallel Resonance

We know from the previous series resonance tutorial that resonance takes place when $V_L = -V_C$ and this situation occurs when the two reactances are equal, $X_L = X_C$. The admittance of a parallel circuit is given as:

$$Y = G + B_L + B_C$$

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

or

$$Y = \frac{1}{R} + \frac{1}{2\pi f L} + 2\pi f C$$

Parallel Resonance

In a **parallel resonant circuit**, the **resonant frequency, f_r** point can be calculated as follows.

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

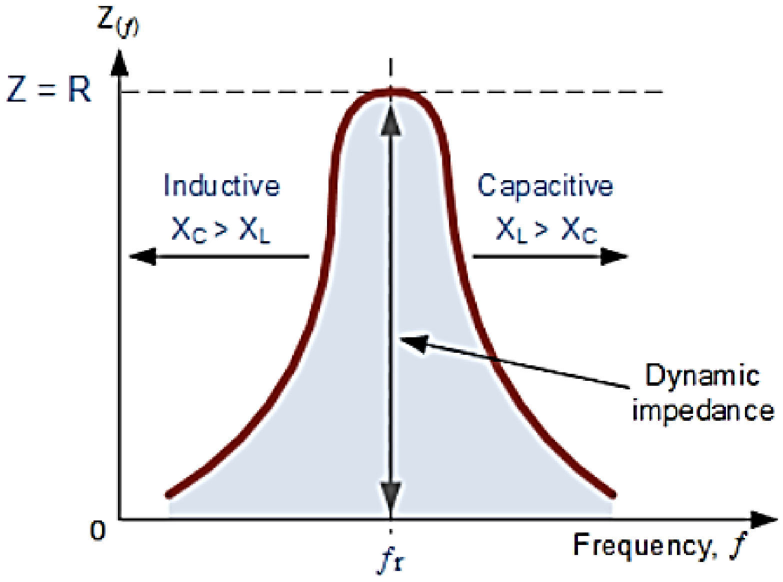
$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} \text{ (Hz)} \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \text{ (rads)}$$

Parallel Resonance

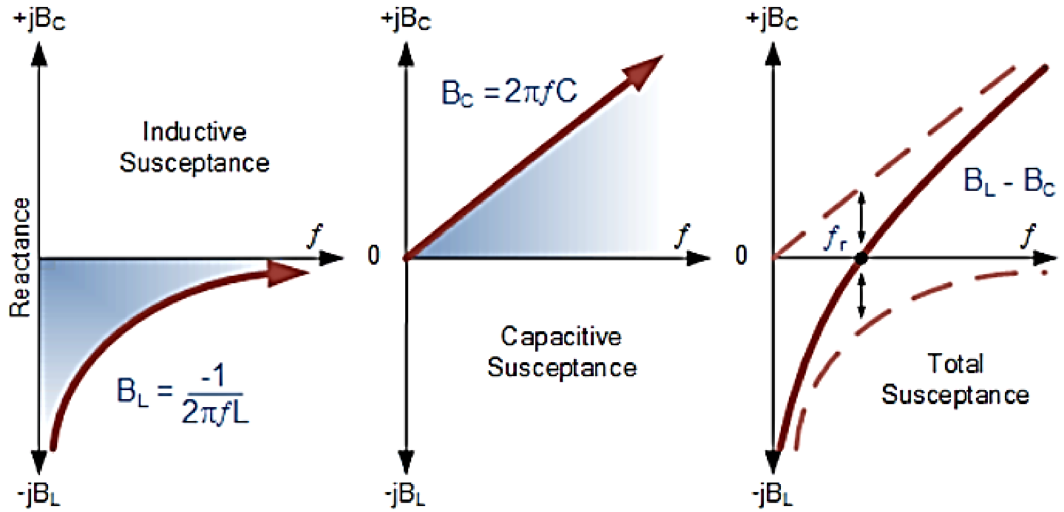
Impedance in a Parallel Resonance Circuit



Parallel Resonance

Note that if the parallel circuits impedance is at its maximum at resonance, then consequently, the circuits admittance must be at its minimum and one of the characteristics of a parallel resonance circuit is that admittance is very low limiting the circuits current. Unlike the series resonance circuit, the resistor in a parallel resonance circuit has a damping effect on the circuits bandwidth making the circuit less selective.

Susceptance at Resonance



From above, the inductive susceptance, B_L is inversely proportional to the frequency as represented by the hyperbolic curve. The capacitive susceptance, B_C is directly proportional to the frequency and is therefore represented by a straight line. The final curve shows the plot of total susceptance of the parallel resonance circuit versus the frequency and is the difference between the two susceptance's.

Parallel Resonance

We remember that the total current flowing in a parallel RLC circuit is equal to the vector sum of the individual branch currents and for a given frequency is calculated as:

$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L}$$

$$I_C = \frac{V}{X_C} = V.2\pi f C$$

Therefore, I_T = vector sum of $(I_R + I_L + I_C)$

$$I_T = \sqrt{I_R^2 + (I_L + I_C)^2}$$

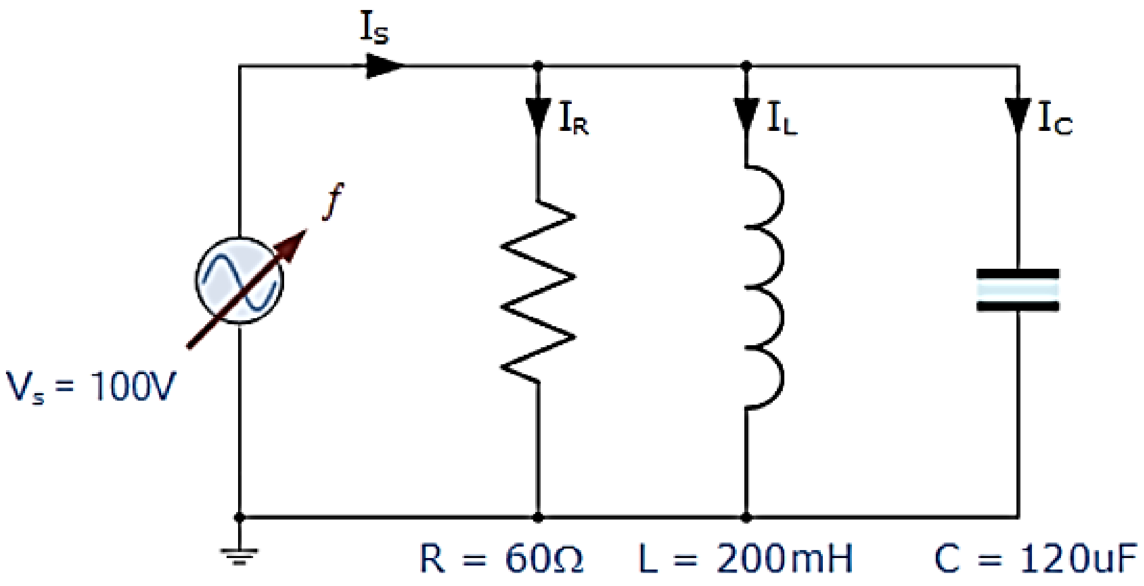
Parallel Resonance

At resonance, currents I_L and I_C are equal and cancelling giving a net reactive current equal to zero. Then at resonance the above equation becomes.

$$I_T = \sqrt{I_R^2 + 0^2} = I_R$$

Parallel Resonance Example No1

A parallel resonance network consisting of a resistor of 60Ω, a capacitor of 120uF and an inductor of 200mH is connected across a sinusoidal supply voltage which has a constant output of 100 volts at all frequencies. Calculate, the resonant frequency, the quality factor and the bandwidth of the circuit, the circuit current at resonance and current magnification.



Parallel Resonance

1. Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2.120.10^{-6}}} = 32.5\text{Hz}$$

2. Inductive Reactance at Resonance, X_L

$$X_L = 2\pi f L = 2\pi.32.5.0.2 = 40.8\Omega$$

3. Quality factor, Q

$$Q = \frac{R}{X_L} = \frac{R}{2\pi f L} = \frac{60}{40.8} = 1.47$$

4. Bandwidth, BW

$$BW = \frac{f_r}{Q} = \frac{32.5}{1.47} = 22\text{Hz}$$

5. The upper and lower -3dB frequency points, f_H and f_L

$$f_L = f_r - \frac{1}{2}BW = 32.5 - \frac{1}{2}(22) = 21.5Hz$$

$$f_H = f_r + \frac{1}{2}BW = 32.5 + \frac{1}{2}(22) = 43.5Hz$$

6. Circuit Current at Resonance, I_T

At resonance the dynamic impedance of the circuit is equal to R

$$I_T = I_R = \frac{V}{R} = \frac{100}{60} = 1.67A$$

7. Current Magnification, I_{mag}

$$I_{MAG} = Q \times I_T = 1.47 \times 1.67 = 2.45A$$

Parallel Resonance

Note that the current drawn from the supply at resonance (the resistive current) is only 1.67 amps, while the current flowing around the LC tank circuit is larger at 2.45 amps. We can check this value by calculating the current flowing through the inductor (or capacitor) at resonance.

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L} = \frac{100}{2\pi.32.5.0.2} = 2.45A$$

Part 2

Passive Filter Circuits

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A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

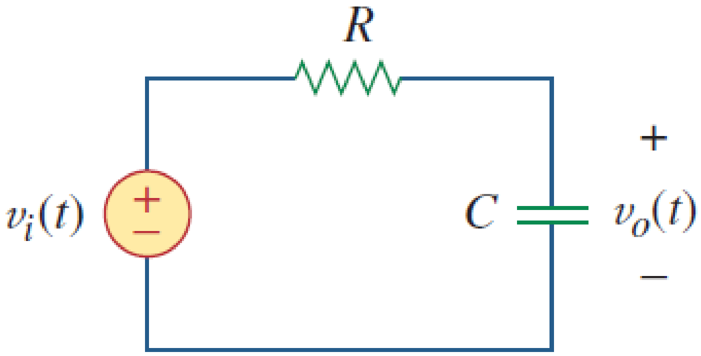
As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies.

Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment.

A filter is a passive filter if it consists of only passive elements R, L, and C. It is said to be an active filter if it consists of active elements (such as transistors and op amps) in addition to passive elements R, L, and C.

Low-Pass Filter (LPF)

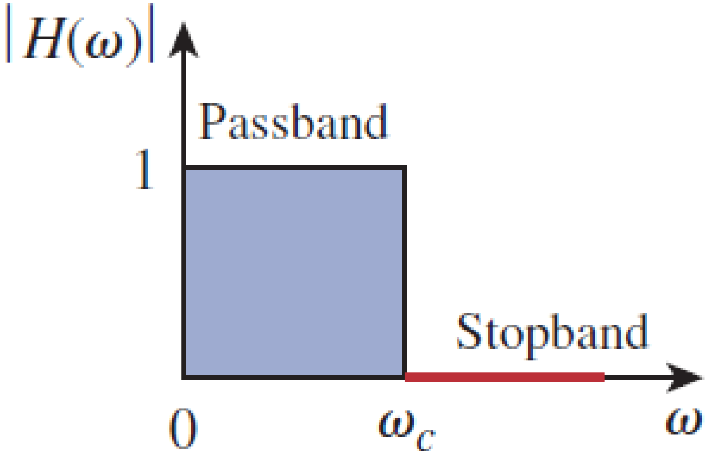
A simple **RC low-pass filter** consists of a resistor and a capacitor connected in series, with the output taken across the capacitor.



The frequency between the pass-and-stop bands is called the cut-off frequency (ω_c). All the signals with frequencies below ω_c are transmitted and all other signals are stopped.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

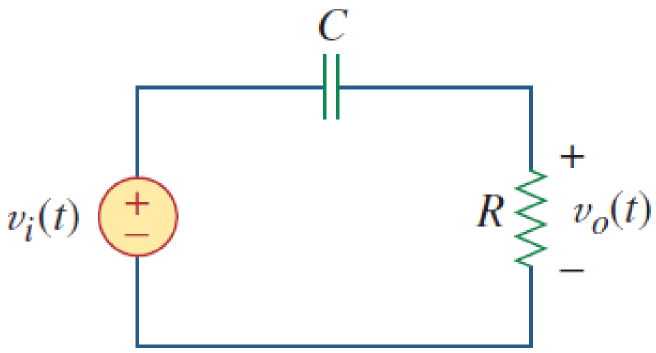


$$f_c = \frac{1}{2\pi RC}\ (Hz)$$

$$\omega_c = 2\pi f_c = \frac{1}{RC}\ (rad)$$

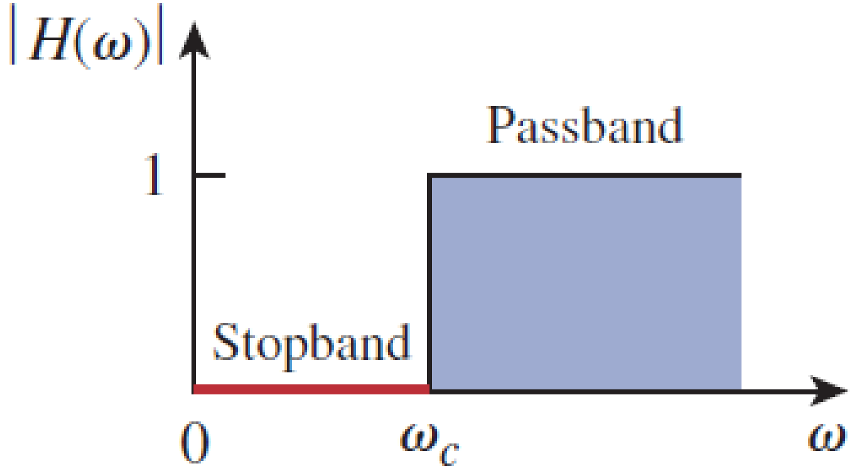
High-Pass Filter (HPF)

A **high-pass filter** allows high frequencies while attenuating low frequencies. The capacitor and resistor positions are swapped compared to the low-pass filter.



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

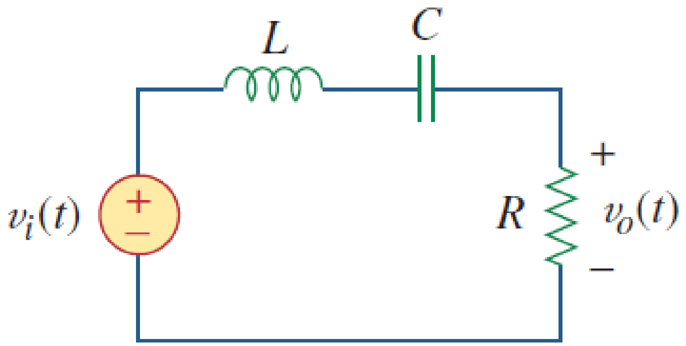


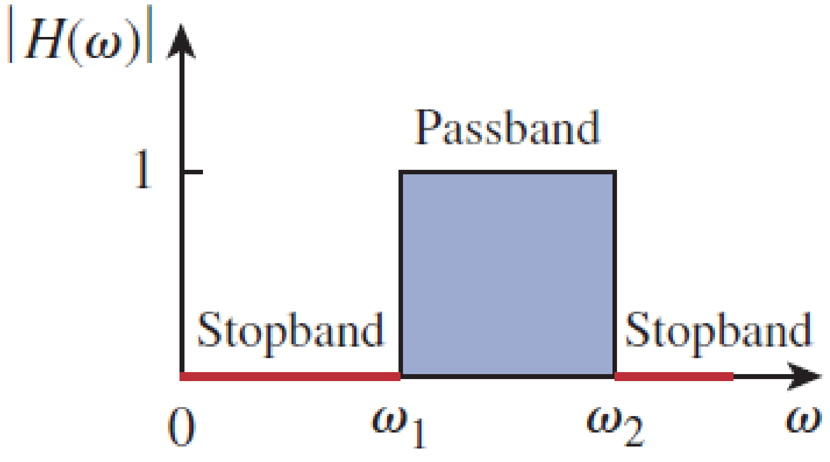
$$f_c = \frac{1}{2\pi RC}\ (Hz)$$

$$\omega_c = 2\pi f_c = \frac{1}{RC}\ (rad)$$

Band-Pass Filter (BPF)

A **band-pass filter** allows only a certain frequency range while attenuating frequencies outside that range. It is formed by combining **HPF** and **LPF**.





$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \ (Hz)$$

Resonant frequency

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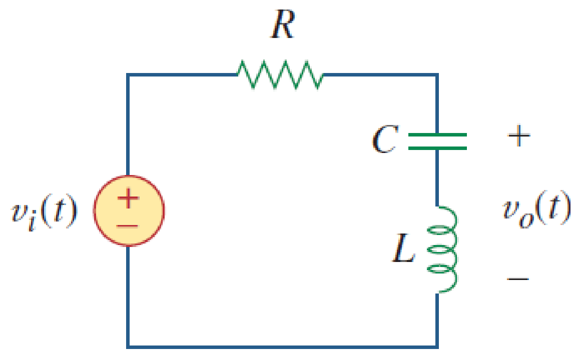
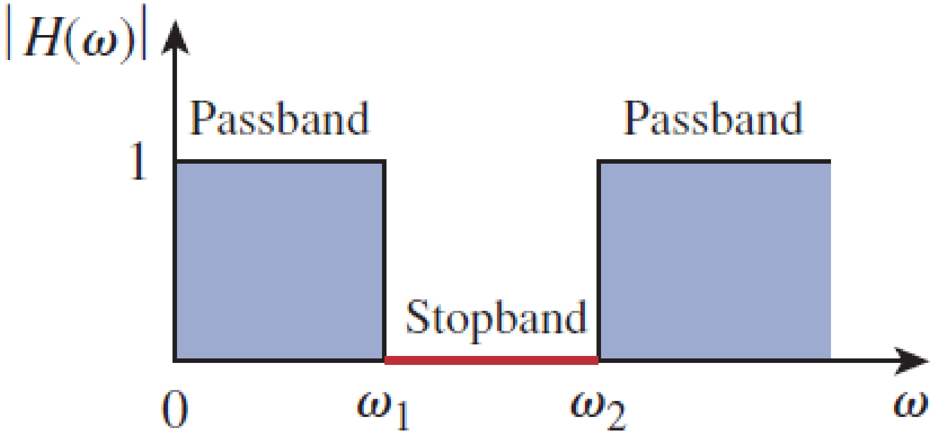
Band-Stop Filter

A band-stop filter blocks a specific range of frequencies while allowing frequencies below and above that range to pass. It is formed by combining a **LPF** and a **HPF** in parallel, so that the unwanted frequency band is attenuated while the remaining frequencies are preserved.





$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

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