University of Diyala

College of Engineering

Department of Power and Electrical Machines Engineering

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Second Year – Bachelor (BSc) – EPE 204

Spring Semester

EPE 204 Electric Circuits Analysis II

Lecture 2

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Lecture 2

Introduction to Transients – Source-Free RC and RL Circuits

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Part 1

Introduction to Transients

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A transient response is the behavior of a circuit immediately after a switching action (ON/OFF, connection/disconnection of a source).

Electrical circuits containing energy storage elements like Capacitors (C) and Inductors (L) cannot change their stored energy instantaneously.

Therefore, the circuit response consists of:

Total Response = Transient Response + Steady-State Response

For source-free circuits, the response is purely transient.

Energy Storage Elements Review

A) Capacitor

Current-voltage relation:

$$i_C = C\frac{d_{v_c}}{dt}$$

Energy stored:

$$\omega_C = \frac{1}{2}Cv^2$$

Capacitor voltage cannot change instantaneously.

B) Inductor

Voltage-current relation:

$$v_L = L\frac{d_{i_L}}{dt}$$

Energy stored:

$$\omega_L = \frac{1}{2}Li^2$$

Inductor current cannot change instantaneously.

Part 2

Source-Free RC Circuit

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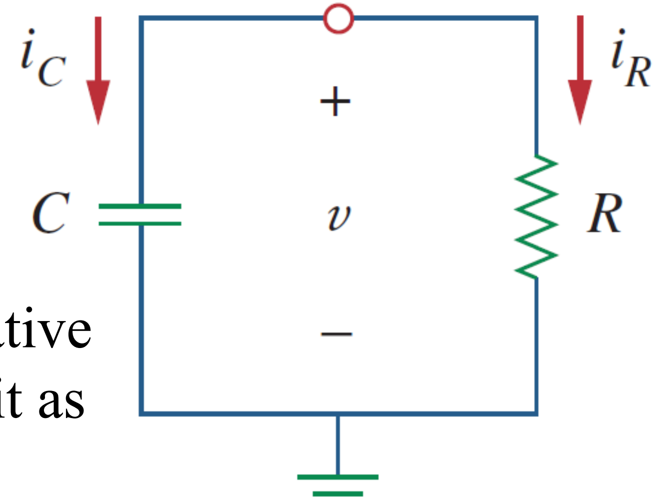
A source-free RC circuit is a first-order circuit containing a resistor and capacitor in which all independent sources are removed, and the response is due only to the energy initially stored in the capacitor. Assume to be the voltage *v(t)* across the capacitor. Since the capacitor is initially charged, we can assume that at time *(t=0)* the initial voltage is $v(o) = V_o$



with the corresponding value of the energy stored as $\omega(0) = \frac{1}{2} C V_0^2$

Applying KCL at the top node of the circuit $i_C + i_R = 0$ And this leads to:

$C\frac{d_v}{dt} + \frac{v}{R} = 0$ or $\frac{d_v}{dt} + \frac{v}{RC} = 0$ This is a 1st DE, since only the first derivative of *v* is involved. To solve it, we rearrange it as

$\frac{d_v}{v} = -\frac{1}{RC}dt$ Integrating both sides ➡ $ln\, v = -\frac{t}{RC} + ln\, A$

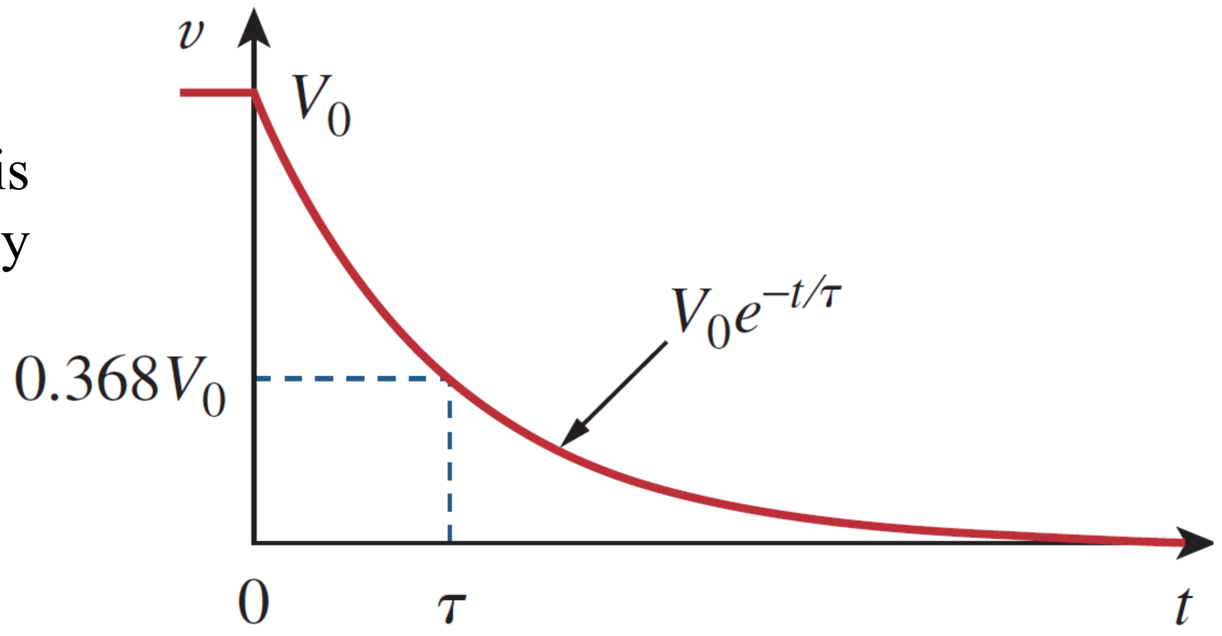
$ln\, A$ is the integration constant

$ln\frac{v}{A} = -\frac{t}{RC}$ Taking powers of *e* leads to ➡ $v(t) = Ae^{-t/RC}$

But from the initial conditions $v(o) = V_o = A$ ➡ $v(t) = V_o e^{-t/RC}$

This shows that the voltage response of the RC circuit is an exponential decay of the initial voltage. Since the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source, it is called the **natural response** of the circuit refers to the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.

The natural response of RC circuit is shown in the right-side figure. Note that as t increases, the voltage decreases toward zero.



The rapidity with which the voltage decreases is expressed in terms of the time constant, denoted by τ.

This implies that at $t = \tau$

$$V_o e^{-\tau/RC} = V_o e^{-1} = 0.36788\,\text{V}$$

Values of $v(t)/V_0 = e^{-t/\tau}$.

t	$v(t)/V_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

The power dissipated in the resistor is

$$p(t) = vi_R = \frac{V_0^2}{R}e^{-2t/\tau}$$

The energy absorbed by the resistor up to time t is

$$w_R(t) = \int_0^t p(\lambda)d\lambda = \int_0^t \frac{V_0^2}{R}e^{-2\lambda/\tau}d\lambda$$

$$= -\frac{\tau V_0^2}{2R}e^{-2\lambda/\tau}\Big|_0^t = \frac{1}{2}CV_0^2(1 - e^{-2t/\tau}), \qquad \tau = RC$$

Notice that as $t \to \infty$, $w_R(\infty) \to \frac{1}{2}CV_0^2$, which is the same as $w_C(0)$, the energy initially stored in the capacitor. The energy that was initially stored in the capacitor is eventually dissipated in the resistor.

Part 3

Source-Free RL Circuit

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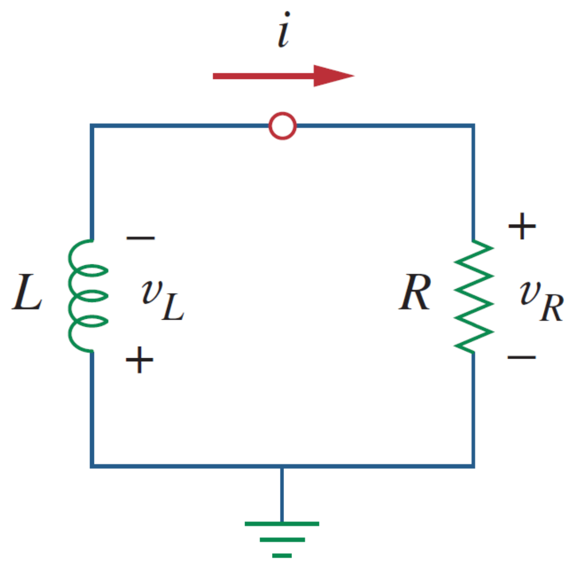
A source-free RL circuit is a first-order circuit containing a resistor and an inductor in which all independent sources are removed, and the response is due only to the energy initially stored in the inductor. Assume to be the current *i(t)* through the inductor. At *(t=0)* we assume that the inductor has an initial current $i(o) = I_o$



with the corresponding value of the energy stored as $\omega(0) = \frac{1}{2} L I_0^2$

Applying KVL around the loop $v_L + v_R = 0$ And this leads to:

$L\frac{d_i}{dt} + Ri = 0$ or $\frac{d_i}{dt} + \frac{R}{L} i = 0$ Rearranging terms and integrating gives

$ln\, i(t) - ln\, I_o = -\frac{Rt}{L} + 0$ or $ln \frac{i(t)}{I_o} = -\frac{Rt}{L}$ Taking powers of *e* leads to

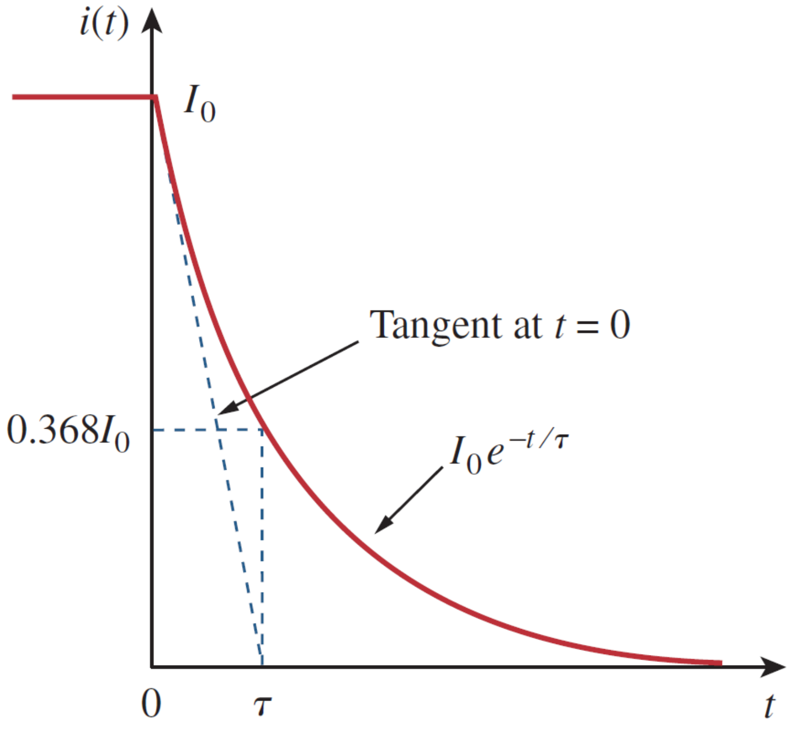
$i(t) = I_o e^{-Rt/L}$

This shows that the natural response of the RL circuit is an exponential decay of the initial current. The current response is shown in the right-side figure. It is evident from $i(t) = I_o e^{-Rt/L}$ that the time constant for the RL circuit is τ

$$\tau = \frac{R}{L}$$

with τ again having the unit of seconds. Thus,

$$\boxed{i(t) = I_0 e^{-t/\tau}}$$



The power dissipated in the resistor is

$$p = v_R i = I_0^2 R e^{-2t/\tau}$$

The energy absorbed by the resistor is

$$w_R(t) = \int_0^t p(\lambda) d\lambda = \int_0^t I_0^2 e^{-2\lambda/\tau} \, d\lambda = -\frac{\tau}{2} I_0^2 R e^{-2\lambda/\tau} \Big|_0^t, \qquad \tau = \frac{L}{R}$$

or

$$w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

Note that as $t \to \infty$, $w_R(\infty) \to \frac{1}{2} L I_0^2$, which is the same as $w_L(0)$, the initial energy stored in the inductor

Again, the energy initially stored in the inductor is eventually dissipated in the resistor.