University of Diyala

College of Engineering

Department of Power and Electrical Machines Engineering

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Second Year – Bachelor (BSc) – EPE 204

Spring Semester

EPE 204 Electric Circuits Analysis II

Lecture 3

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Lecture 3

Step Response of an RC and an RL Circuits

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The Step Function

A **step input** is a sudden change in voltage or current from zero to a constant value.

Mathematically, the unit step function is:

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

When the dc source of an *RC/RL* circuit is suddenly applied, the voltage or current source can be modelled as a step function, and the response is known as a *step response.*

The step response of a circuit is the behavior of it when the excitation is the step function, which may be a voltage or a current source.

Part 1

Step Response of an RC Circuits

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Consider the RC circuit in Fig. (a) which can be replaced by the circuit in Fig. (b), where V_s is a constant dc voltage source. Again, select the capacitor voltage as the circuit response to be determined.

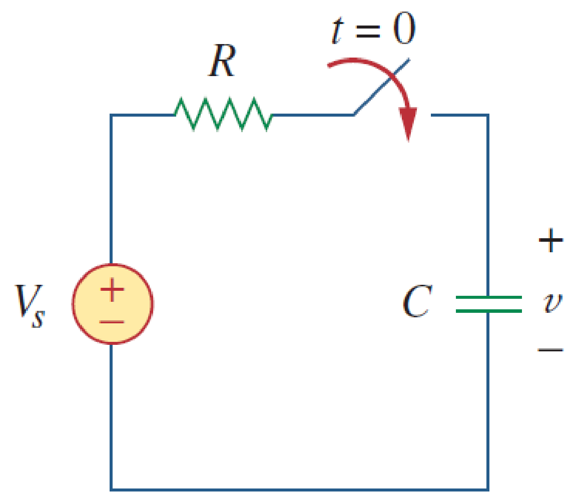
Assume an initial voltage on the capacitor V_0 , although this is not necessary for the step response. Since the voltage of a capacitor cannot change at once,

$$v(0^-) = v(0^+) = V_0$$

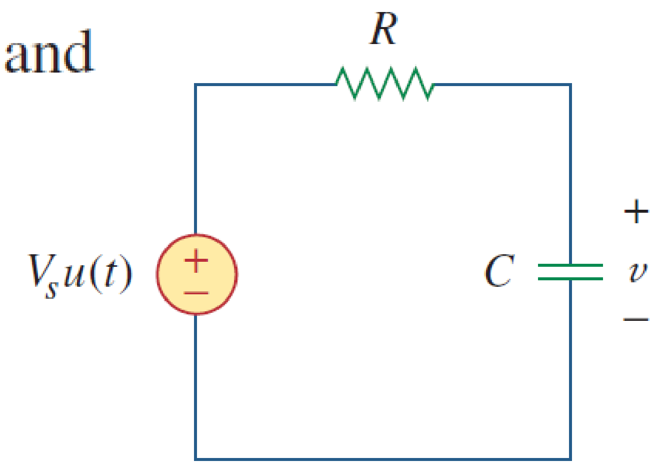
where $v(0^-)$ is the voltage across the capacitor just before switching and $v(0^+)$ is its voltage immediately after switching.

After applying KCL, we obtain:

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \qquad t > 0$$



(a)



(b)

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The first way to break the complete response into a "natural response" and a "forced response"

$$\text{Complete response} = \underbrace{\text{natural response}}_{\text{stored energy}} + \underbrace{\text{forced response}}_{\text{independent source}}$$

or

$$v = v_n + v_f$$

where

$$v_n = V_o e^{-t/\tau}$$

and

$$v_f = V_s(1 - e^{-t/\tau})$$

The second way to break the complete response into a “transient response” and a “steady-state

$$\text{Complete response} = \underbrace{\text{transient response}}_{\text{temporary part}} + \underbrace{\text{steady-state response}}_{\text{permanent part}}$$

or

$$v = v_t + v_{ss}$$

where

$$v_t = (V_o - V_s)e^{-t/\tau}$$

and

$$v_{ss} = V_s$$

The transient response is the circuit’s temporary response that will die out with time.

The steady-state response is the behavior of the circuit a long time after an external excitation is applied.

The **first decomposition** of the complete response is in terms of **the source of the responses**, while the **second decomposition** is in terms of **the permanency of the responses**. Under certain conditions, the natural response and transient response are the same. The same can be said about the forced response and steady-state response.

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

Thus, to find the step response of an RC circuit requires three things:

1. The initial capacitor voltage $v(0)$.
2. The final capacitor voltage $v(\infty)$.
3. The time constant τ.

Part 2

Step Response of an RL Circuits

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Consider the RL circuit in Fig.(a), which may be replaced by the circuit in Fig.(b). Again, our goal is to find the inductor current *i* as the circuit response. Let the response be the sum of the transient response and the steady-state response,

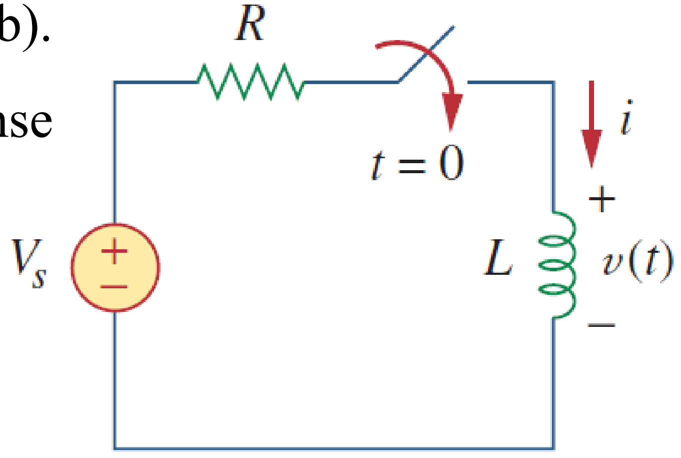
$$i = i_t + i_{ss}$$

We know that the transient response is always a decaying exponential, that is,

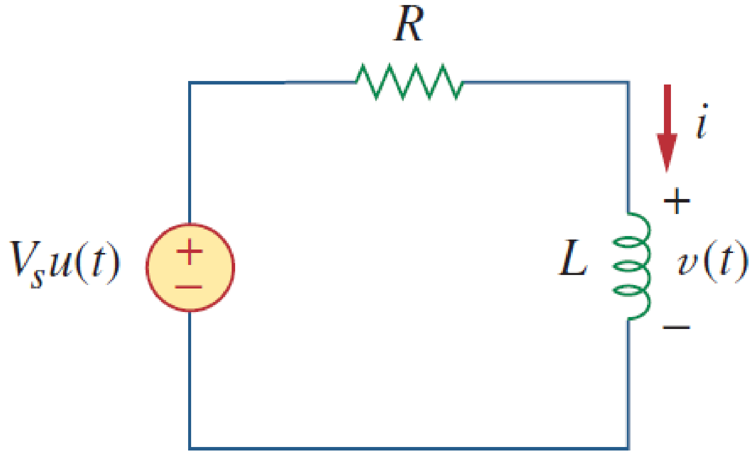
$$i_t = Ae^{-t/\tau}, \qquad \tau = \frac{L}{R}$$

where A is a constant to be determined.

The steady-state response is the value of the current a long time after the switch in Fig.(a) is closed.



(a)



(b)

We know that the transient response essentially dies out after five-time constants. At that time, the inductor becomes a short circuit, and the voltage across it is zero. The entire source voltage V_s appears across R. Thus, the steady-state response is

$$i_{ss} = \frac{V_s}{R}$$

$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

We now determine the constant A from the initial value of i. Let I_0 be the initial current through the inductor, which may come from a source other than V_s. Since the current through the inductor cannot change instantaneously,

$$i(0^+) = i(0^-) = I_0$$

Thus, at $t = 0$,

$$I_0 = A + \frac{V_s}{R}$$

From this, we obtain A as

$$A = I_0 - \frac{V_s}{R}$$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

This is the complete response of the RL circuit.

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

Thus, to find the step response of an RL circuit requires three things:

1. The initial inductor current $i(0)$ at $t = 0$.
2. The final inductor current $i(\infty)$.
3. The time constant τ.