University of Diyala

College of Engineering

Department of Power and Electrical Machines Engineering

Academic Year 2025-2026

Second Year – Bachelor (BSc) – EPE 204

Spring Semester

EPE 204 Electric Circuits Analysis II

Lecture 5

Assist. Lect. Ibrahim I. Ibrahim

Lecture 5

The Source-Free Series and Parallel RLC Circuit

Assist. Lect. Ibrahim I. Ibrahim

A **source-free RLC circuit** is a circuit where:

- All **independent sources are removed (set to zero)**
- The response depends only on **initial stored energy**
  - Capacitor → stored electric energy 
  - Inductor → stored magnetic energy 

👉 This leads to a **natural response** (transient response only).

An understanding of the natural response of the series and parallel **RLC** circuit is a necessary background for future studies in filter design and communications networks.

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Part 1

The Source-Free Series RLC Circuit

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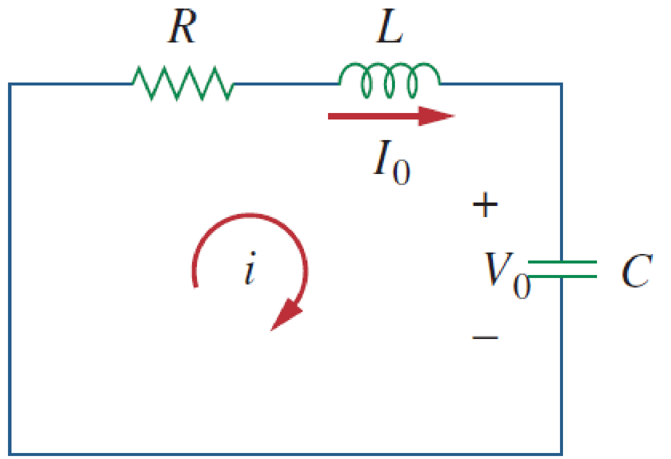

A series connection of RLC with initial energy in L and/or C.

Governing Equation $v_R + v_L + v_C = 0$ and substitute the following

$$v_R = iR \qquad v_L = L\frac{di}{dt} \qquad v_C = \frac{1}{C}\int i\ dt$$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\ dt = 0$$ Differentiate to eliminate the integral:

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$ Thus, the standard form will be

$$\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0}$$

Now, define new parameters:

Damping factor /Neper frequency (Np/s) $$\alpha = \frac{R}{2L}$$

Undamped Natural frequency (rad/s) $$\omega_o = \frac{1}{\sqrt{LC}}$$

Characteristic Equation

$$s^2 + 2\alpha s + \omega_o^2$$

The roots s_1 and s_2 are called natural frequencies (Np/s)

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Types of Responses

1. Overdamped ($\alpha > \omega_0$)

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

- Two real roots

2. Critically Damped ($\alpha = \omega_0$)

$$i(t) = (A_1 + A_2 t)e^{-\alpha t}$$

$$s_1 = s_2 = -\alpha$$

- Repeated root

3. Underdamped ($\alpha < \omega_0$)

$$i(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

- Complex roots

$$s_1 = -\alpha + j\omega_d$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$s_2 = -\alpha - j\omega_d$$

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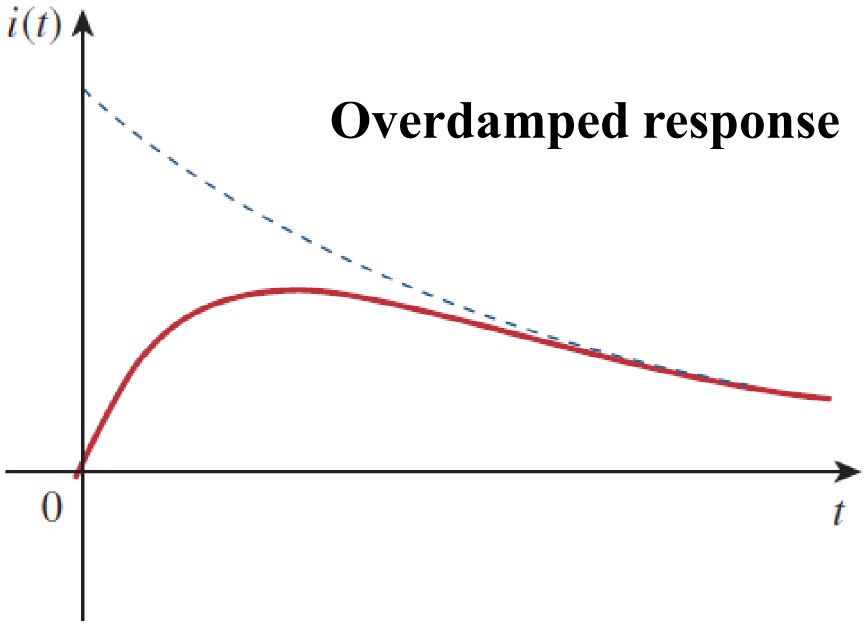
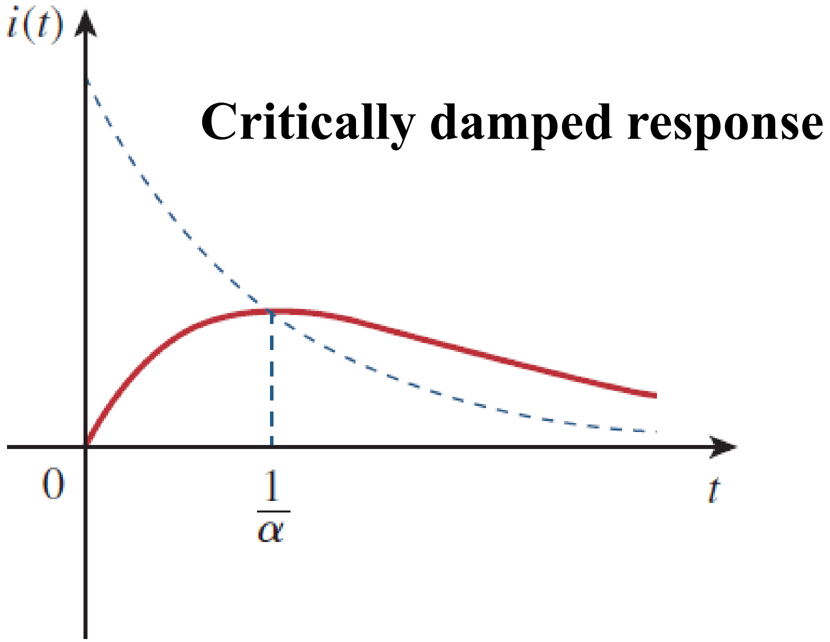
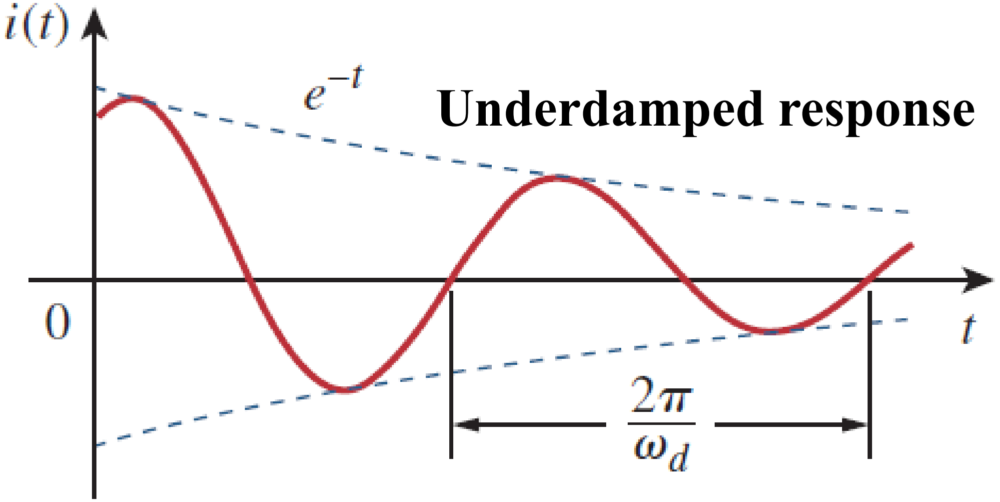

i(t)
Overdamped response
0
t


i(t)
Critically damped response
0
$\frac{1}{\alpha}$
t


i(t)
e^{-t}
Underdamped response
0
t
$\frac{2\pi}{\omega_d}$

Example 1: Calculate the roots of the RLC series circuit knowing that $R = 40\Omega, L = 4H$ and $C = 0.25F$. Is the natural response overdamped, underdamped, or critically damped?

We first calculate

$$\alpha = \frac{R}{2L} = \frac{40}{2(4)} = 5, \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times \frac{1}{4}}} = 1$$

Since $\alpha > \omega_o$ this means the response is overdamped, and this leads to:

The roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1}$$

or

$$s_1 = -0.101, \qquad s_2 = -9.899$$

This is also evident from the fact that the roots are real and negative.

Part 2

The Source-Free Parallel RLC Circuit

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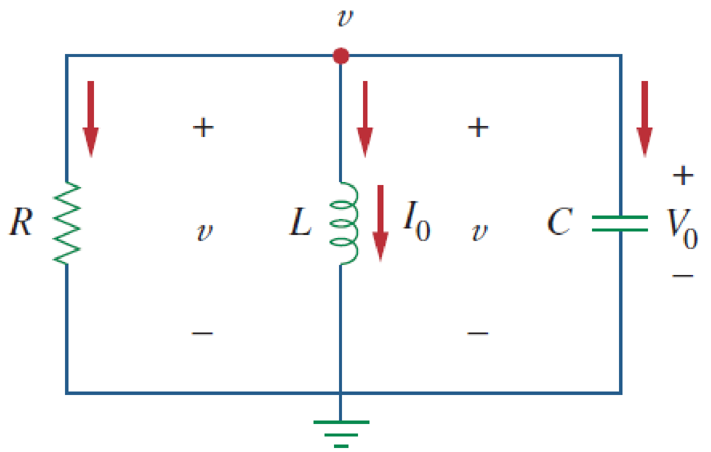
A parallel connection of RLC with initial energy in L and/or C.



Governing Equation $i_R + i_L + i_C = 0$ and substitute the following

$$i_R = \frac{v}{R} \qquad i_L = \frac{1}{L}\int v\ dt \qquad i_C = C\frac{dv}{dt}$$

$$\frac{v}{R} + \frac{1}{L}\int v\ dt + C\frac{dv}{dt} = 0$$ Differentiate to eliminate the integral:

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$ Thus, the standard form will be

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0}$$

Now, define new parameters:

Damping factor /Neper frequency (Np/s) $$\alpha = \frac{1}{2RC}$$

Undamped Natural frequency (rad/s) $$\omega_o = \frac{1}{\sqrt{LC}}$$

Characteristic Equation

$$s^2 + 2\alpha s + \omega_o^2$$

The roots s_1 and s_2 are called natural frequencies (Np/s)

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Types of Responses

1. Overdamped ($\alpha > \omega_0$)

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

- Two real roots

2. Critically Damped ($\alpha = \omega_0$)

$$v(t) = (A_1 + A_2 t)e^{-\alpha t}$$

$$s_1 = s_2 = -\alpha$$

- Repeated root

3. Underdamped ($\alpha < \omega_0$)

$$v(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

- Complex roots

$$s_1 = -\alpha + j\omega_d$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$s_2 = -\alpha - j\omega_d$$

Example 2: Calculate the roots of the RLC parallel circuit knowing that R = 2Ω, L = 1H and C = 0.5F . Is the natural response overdamped, underdamped, or critically damped?

We first calculate $$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 2 \times 0.5} = \frac{1}{2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1$$

Since $\alpha < \omega_o$ this means the response is underdamped, and this leads to:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{1 - 0.25} = 0.866$$

$$s_{1,2} = -0.5 \pm j\omega_d$$

Comparison: Series vs Parallel RLC

Feature	Series RLC	Parallel RLC
Variable	$i(t)$	$v(t)$
Damping factor	$\alpha = \frac{R}{2L}$	$\alpha = \frac{1}{2RC}$
Governing law	KVL	KCL
Energy loss	R	R