UNIVERSITY OF DIYALA
COLLEGE OF ENGINEERING
DEPARTMENT OF THE ELECTRICAL POWER AND MACHINES ENGINEERING

Second Stage

ELECTRICAL MACHINES

Lecture: 2

Lecturer
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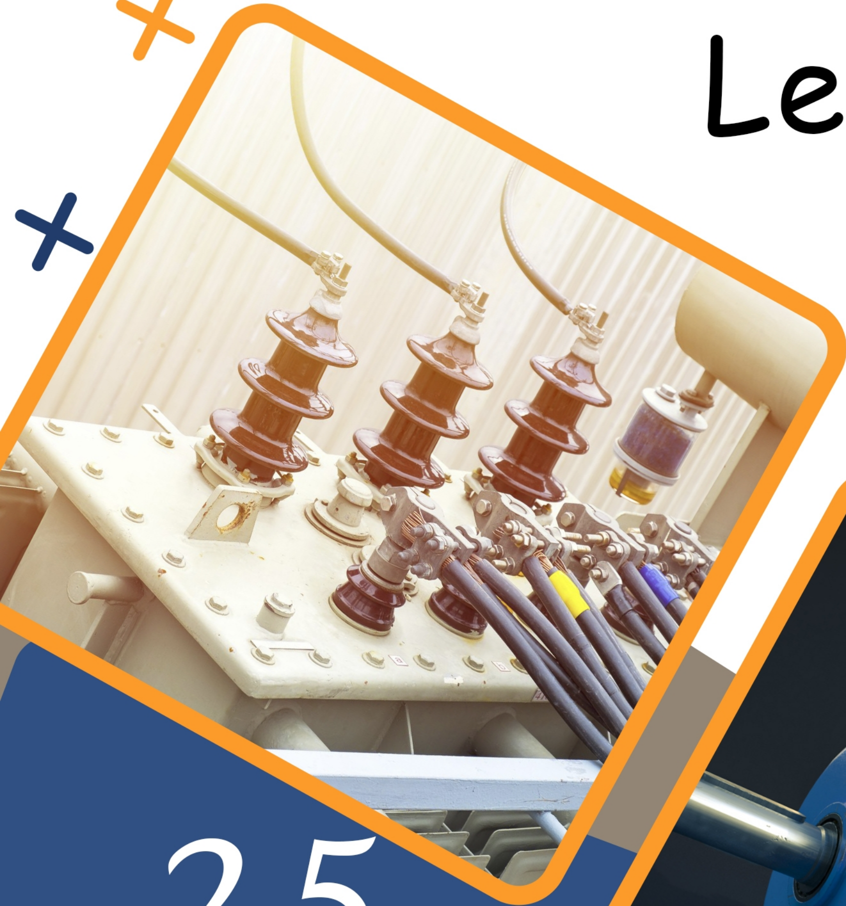

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Generated E.M.F. or E.M.F. Equation of a Generator:

Let:

Φ = flux/pole in weber
Z = total number of armature conductors
Z = No. of slots $\times$ No. of conductors/slot
P = No. of generator poles
A = No. of parallel paths in armature
N = armature rotation in revolutions per minute $(r.p.m.)$
E = e.m.f. induced in any parallel path in armature

For a simplex wave-wound generator:

No. of parallel paths = 2
No. of conductors (in series) in one path = $Z/2$

$$\therefore \text{ E.M.F. generated/path } = \frac{\Phi PN}{60} \times \frac{Z}{2} = \frac{\Phi ZPN}{120} \text{ volt}$$

For a simplex lap-wound generator:

No. of parallel paths = P
No. of conductors (in series) in one path = $\frac{Z}{P}$

$$\therefore \text{ E.M.F. generated/path } = \frac{\Phi PN}{60} \times \frac{Z}{P} = \frac{\Phi ZN}{60} \text{ volt}$$

$$\text{In general generated e.m.f. } E_g = \frac{\Phi ZN}{60} \times \frac{P}{A} \text{ volt}$$

Where:
$A = 2$ (for simplex wave-winding)
$A = P$ (for simplex lap-winding)

Example 1: A four-pole generator, having wave-wound armature winding has 51 slots, each slot containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 $r.p.m$ assuming the flux per pole to be 7.0 mWb?

Solution:

$$E_g = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \text{ volt}$$

Here, $\Phi = 7 \times 10^{-3}$Wb ; $Z = 51 \times 20 = 1{,}020$; $A = 2$; $P = 4$;
$N = 1{,}500\ r.p.m.$

$$\therefore \quad E_g = \frac{7 \times 10^{-3} \times 1{,}020 \times 1{,}500}{60} \times \left(\frac{4}{2}\right) = 357\text{V}$$

Example 2: An 8-pole D.C. generator has 500 armature conductors, and a useful flux of 0.05Wb per pole. What will be the e.m.f. generated if it is lap-connected and runs at 1,200 $r.p.m$? What must be the speed at which it is to be driven produce the same e.m.f. if it is wave-wound?

Solution: With lap-winding, $P = A = 8$

$$E = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right)$$

$$E = \frac{0.05\times500\times1{,}200}{60}\times\frac{8}{8} = 500\text{volt}$$

If it is wave-wound, $P = 8$, $A = 2$,

And $E = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right)$ volt

$$E = \frac{0.05\times500\times N}{60}\times\frac{8}{2}$$

For $E = 500\text{volt}$, $N = 300\ r.p.m.$

Hence, with wave-winding, it must be driven at 300 $r.p.m.$ to generate 500volt.

Example 3: In a 120V compound generator, the resistances of the armature, shunt and series windings are 0.06Ω, 25Ω and 0.04Ω respectively. The load current is 100A at 120V. Find the induced e.m.f. and the armature current when the machine is connected as (*i*) long-shunt and as (*ii*) short-shunt.

Solution:

(*i*) Long Shunt Figure (1-a)

$$I_{sh} = \frac{120}{25} = 4.8\text{A}\,;\ I = 100\text{A}\,;\ I_a = 104.8\text{A}$$

Voltage drop in series winding

$= 104.8 \times 0.04 = 4.19\text{V}$

Armature voltage drop

$= 104.8 \times 0.06 = 6.29\text{V}$

$\therefore\ E_g = 120 + 4.19 + 6.29 = 130.5\text{V}$

(*ii*) Short Shunt Figure (1-b)

Voltage drop in series winding

$= 100 \times 0.04 = 4\text{V}$

Voltage across shunt winding

$= 120 + 4 = 124\text{V}$

$\therefore\ I_{sh} = \frac{124}{25} \cong 5\text{A}$

$I_a = 100 + 5 = 105\text{A}$

Armature voltage drop

$= 105 \times 0.06 = 6.3\text{V}$

$E_g = 120 + 6.3 + 4 = 130.3\text{V}$

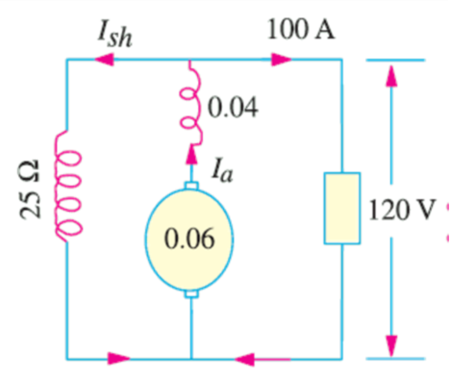

Figure (1-a)

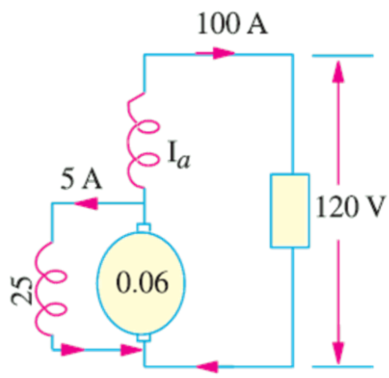

Figure (1-b)

Example 4: 4-pole, long-shunt lap-wound generator supplies 25kW at a terminal voltage of 500V. The armature resistance is 0.03Ω, series field resistance is 0.04Ω and shunt field resistance is 200Ω. The brush drop may be taken as 1.0V. Determine the e.m.f. generated.
Calculate also the No. of conductors if the speed is 1,200 $r.p.m$ and flux per pole is 0.02Wb. Neglect armature reaction.

Solution: $I = \frac{25,000}{500} = 50A$; $I_{sh} = \frac{500}{200} = 2.5A$

$$I_a = I + I_{sh} = 50 + 2.5 = 52.5A$$

Series field drop $= 52.5 \times 0.04 = 2.1V$
Armature drop $= 52.5 \times 0.03 = 1.575V$
Brush drop $= 2 \times 1 = 2V$
Generated e.m.f.

$$E_G = 500 + 2.1 + 1.575 + 2 = 505.67V$$

Now $E_g = \frac{\Phi Z N}{60}\left(\frac{P}{A}\right)$

$$505.67 = \frac{0.02 \times Z \times 1,200}{60}\left(\frac{4}{4}\right) \Rightarrow\Rightarrow Z = 1,264$$

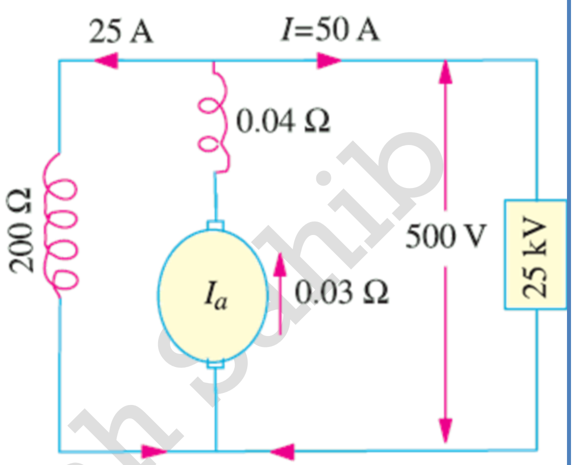

Figure (2)

Example 5: A 4-pole lap-connected armature of a D.C. shunt generator is required to supply the loads connected in parallel:
(1) 5kW Geyser at 250V ; (2) 2.5kW Lighting load also at 250V.
The Generator has an armature resistance of 0.2Ω and a field resistance of 250Ω. The armature has 120 conductors in the slots and runs at 1,000 $r.p.m$. Allowing 1V per brush for contact drops and neglecting friction, find Flux per pole.
Solution:

Geyser current $= \frac{5,000}{250} = 20A$; Current for Lighting $= \frac{2,500}{250} = 10A$
Total current $= 30A$
Field Current for Generator $= \frac{V}{R_{sh}} = \frac{250}{250} = 1A$
Hence, Armature Current, $I_a = I_L + I_{sh} = 30 + 1 = 31A$
Armature resistance drop $= I_a R_a = 31 \times 0.2 = 6.2V$
Generated e.m.f.,

$$E_g = V + I_a R_a + \text{total brush contact drop} = 250 + 6.2 + 2 = 258.2V$$

For a 4-pole lap-connected armature,
Number of parallel paths = number of poles = 4
The flux per pole is obtained from the emf equation,

$$E_g = \frac{\Phi Z N}{60} \times \frac{P}{A} \Rightarrow\Rightarrow 258.2 = \frac{\Phi \times 120 \times 1,000}{60}\frac{4}{4}$$

$$\Phi = 129.1mWb$$

Iron Loss in Armature:

Due to the rotation of the iron core of the armature in the magnetic flux of the field poles, there are some losses taking place continuously in the core and are known as Iron Losses or Core Losses. Iron losses consist of (*i*) Hysteresis loss and (*ii*) Eddy Current loss.

(*i*) Hysteresis Loss (W_h)

This loss is due to the reversal of magnetisation of the armature core. Every portion of the rotating core passes under N and S pole alternately, thereby attaining S and N polarity respectively. The core undergoes one complete cycle of magnetic reversal after passing under one pair of poles. If P is the number of poles and N, the armature speed in $r.p.m.$, then frequency of magnetic reversals is $f = \frac{PN}{120}$.

The loss depends upon the volume and grade of iron, maximum value of flux density B_{max} and frequency of magnetic reversals. For normal flux densities ($i.e.$ upto 1.5 Wb/m^2), hysteresis loss is given by **Steinmetz** formula. According to this formula,

$$W_h = \eta B_{max}^{1.6} f V \text{ watts}$$

Where:

$V =$ volume of the core in m^3

$\eta =$ Steinmetz hysteresis coefficient

Value of η for:

Good dynamo sheet steel $= 502\mathrm{J/m^3}$, Silicon steel $= 191\mathrm{J/m^3}$,
Hard Cast steel $7{,}040\mathrm{J/m^3}$, Cast steel $750 - 3{,}000\mathrm{J/m^3}$
and Cast iron $= 2700 - 4{,}000\mathrm{J/m^3}$

(*ii*) Eddy Current Loss (W_e)

When the armature core rotates, it also cuts the magnetic flux. Hence, an e.m.f. is induced in the body of the core according to the laws of electromagnetic induction. This e.m.f. though small, sets up large current in the body of the core due to its small resistance. This current is known as eddy current.

The power loss due to the flow of this current is known as eddy current loss. This loss would be considerable if solid iron core were used.

In order to reduce this loss and the consequent heating of the core to a small value, the core is built up of thin laminations, which are stacked and then riveted at right angles to the path of the eddy currents. These core laminations are insulated from each other by a thin coating of varnish.

It is found that eddy current loss We is given by the following relation:

$$W_e = K B_{max}^2 f^2 t^2 V^2 \text{ watt}$$

Where:
B_{max} = maximum flux density
f = frequency of magnetic reversals
t = thickness of each lamination
V = volume of armature core
This loss varies directly as the square of the thickness of laminations, hence it should be kept as small as possible because it reduce the efficiency of the generator and raise the temperature of the core.
Eddy current loss is reduced by using laminated core but hysteresis loss cannot be reduced this way. For reducing the hysteresis loss, those metals are chosen for the armature core which has a low hysteresis coefficient. Generally, special silicon steels such as alloys are used which not only have a low hysteresis coefficient but which also possess high electrical resistivity.

Total Loss in a D.C. Generator:

The various losses occurring in a generator can be sub-divided as follows:

(a) Copper Losses:

(*i*) Armature copper loss $= I_a^2 R_a$

Note: $E_g I_a$ is the power output from armature.
Where: R_a = resistance of armature and interpoles and series field winding etc..
This loss is about 30 to 40% of full-load losses.

(*ii*) Field copper loss. In the case of shunt generators, it is practically constant and $I_{sh}^2 R_{sh}$ (or VI_{sh}). In the case of series generator, it is $= I_{se}^2 R_{se}$.
Where:
R_{se} = resistance of the series field winding.
This loss is about 20 to 30% of F.L. losses.

(*iii*) The loss due to brush contact resistance. It is usually included in the armature copper loss.

(b) Magnetic Losses:

(also known as iron or core losses),

(*i*) hysteresis loss, $W_h \propto B_{max}^{1.6} f$

(*ii*) eddy current loss, $W_e \propto B_{max}^2 f^2$

These losses are practically constant for shunt and compound-wound generators, because in their case, field current is approximately constant.
Both these losses total up to about 20 to 30% of F.L. losses.

(c) Mechanical Losses:

These consist of:

(*i*) friction loss at bearings and commutator.

(*ii*) air-friction or windage loss of rotating armature.

These are about 10 to 20% of F.L. Losses.

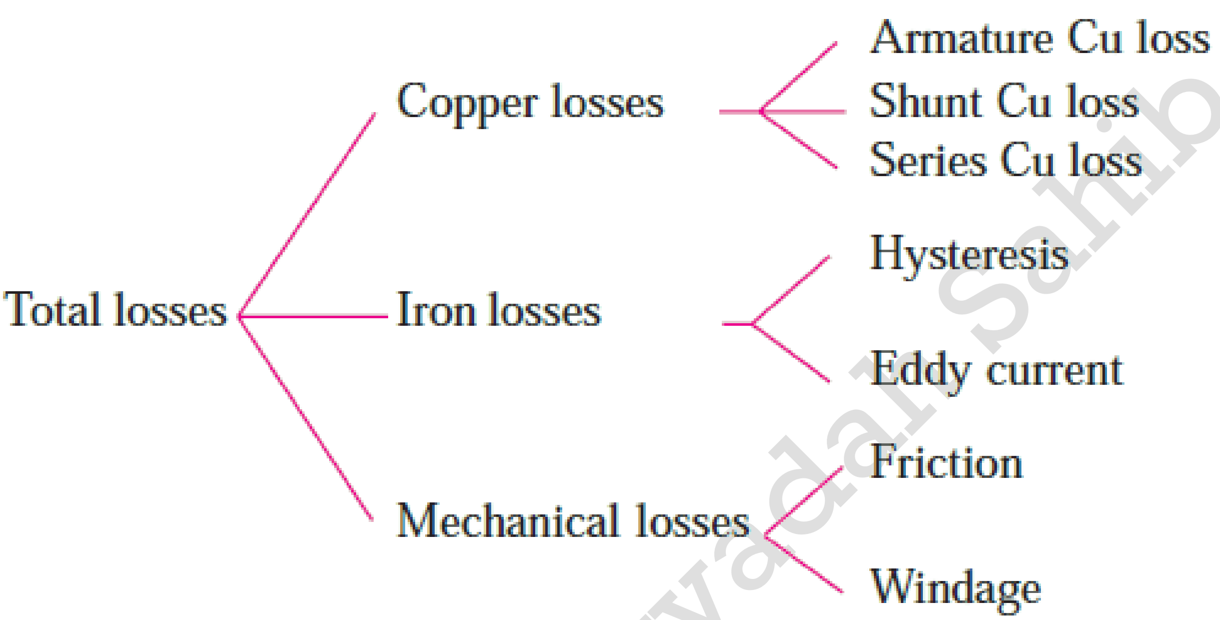

Figure (3)

Stray Losses:

Usually, magnetic and mechanical losses are collectively known as **Stray Losses**. These are also known as rotational losses for obvious reasons.

Constant or Standing Losses:

As said above, field Cu loss is constant for shunt and compound generators. Hence, stray losses and shunt Cu loss are constant in their case. These losses are together known as standing or constant losses W_c.

Hence, for shunt and compound generators,

$$\text{Total loss} = \text{armature copper loss} + W_c = I_a^2 R_a + W_c = (I + I_{sh})^2 R_a + W_c$$

Armature Cu loss $I_a^2 R_a$ is known as variable loss because it varies with the load current.

$$\text{Total loss} = \text{variable loss} + \text{constant losses } W_c$$

Power Stages:

Various power stages in the case of a D.C. generator are shown Figure (4):

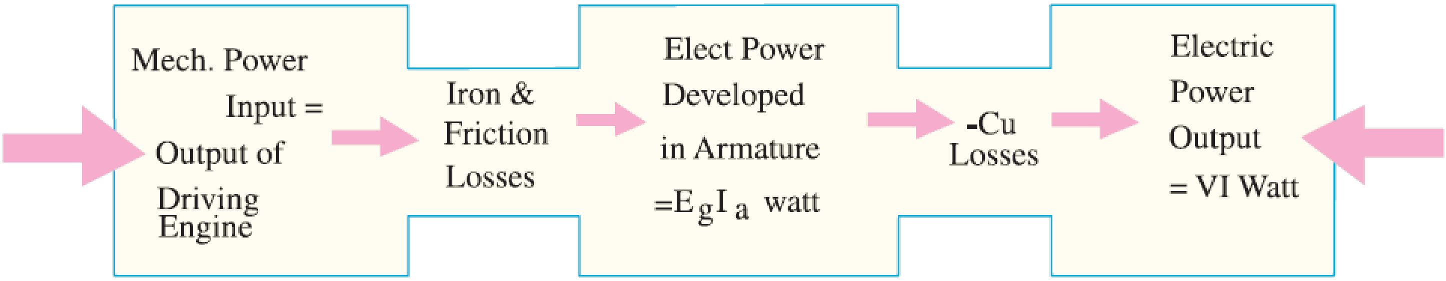

Figure (4)

Following are the three generator efficiencies:

(1) Mechanical Efficiency:

$$\eta_m = \frac{B}{A} = \frac{\text{total watts generated in armature}}{\text{total watts generated in armature}} = \frac{E_g I_a}{\text{output of driving engine}}$$

(2) Electrical Efficiency:

$$\eta_e = \frac{C}{B} = \frac{\text{watts available in load circuit}}{\text{total watts generated}} = \frac{VI}{E_g I_a}$$

(3) Overall or Commercial Efficiency:

$$\eta_c = \frac{C}{A} = \frac{\text{watts available in load circuit}}{\text{mechanical power supplied}}$$

It is obvious that overall efficiency $\eta_c = \eta_m \times \eta_e$. For good generators, its value may be as high as 95%.

Note: Unless specified otherwise, commercial efficiency is always to be understood.

Condition for Maximum Efficiency:

Generator output $= VI$

Generator input = output + losses

Generator input $= VI + I_a^2 R_a + W_c = VI + (I + I_{sh})^2 R_a + W_c \qquad (\because\ I_a + I_{sh})$

However, if I_{sh} is negligible as compared to load current, then $(I_a = I[\text{approx}])$.

$$\therefore\ \eta = \frac{\text{output}}{\text{input}} = \frac{VI}{VI + I_a^2 R_a + W_c} = \frac{VI}{VI + I^2 R_a + W_c} \qquad (\because\ I_a + I_{sh})$$

$$\frac{1}{1 + \left(\frac{IR_a}{V} + \frac{W_c}{VI}\right)}$$

Now, efficiency is maximum when denominator is minimum i.e. when

$$\frac{d}{dI}\left(\frac{IR_a}{V} + \frac{W_c}{VI}\right) = 0 \quad \text{or} \ I^2 R_a = W_c$$

Hence, generator efficiency is maximum when

Variable loss = constant loss

The load current corresponding to maximum efficiency is given by the relation.

$$I^2R_a = W_c \quad \text{or } I = \sqrt{\frac{W_c}{R_a}}$$

Example 6: A 10kW, 250V, D.C, 6-pole shunt generator runs at 1,000 $r.p.m$ when delivering full-load. The armature has 534 lap-connected conductors. Full-load Cu loss is 0.64kW. The total brush drop is 1V. Determine the flux per pole. Neglect shunt current.

Solution:

Since shunt current is negligible, there is no shunt Cu loss. The copper loss occurs in armature only.

$$I = I_a = \frac{10{,}000}{250} = 40\text{A}$$

$$I_a^2R_a = \text{Arm. Cu loss} \quad or\ 40^2 \times R_a = 0.64 \times 10^3 \quad ; \quad R_a = 0.4\Omega$$

$$I_aR_a\text{drop} = 0.4 \times 4 = 16\text{V}$$

Brush drop

$$= 2 \times 1 = 2\text{V}$$

$$\therefore \quad \text{Generated e.m.f.}$$

$$E_g = 250 + 16 + 1 = 267\text{V}$$

$$E_g = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \quad \text{volt}$$

$$\Rightarrow \quad 267 = \frac{\Phi \times 534 \times 1{,}000}{60}\left(\frac{6}{6}\right)$$

$$\Rightarrow \quad \Phi = 30 \times 10^{-3}\text{Wb} = 30\text{mWb}$$

Example 7: A shunt generator delivers 195A at terminal p.d. of 250V. The armature resistance and shunt field resistance are 0.02Ω and 50Ω respectively. The iron and friction losses equal 950W. Find:

(a) E.M.F. generated (b) Cu losses (c) output of the prime motor

(d) commercial, mechanical and electrical efficiencies.

Solution: (a) $I_{sh} = \frac{250}{50} = 5\text{A} \quad ; \quad I_a = 195 + 5 = 200\text{A}$

Armature voltage drop $= I_aR_a = 200 \times 0.02 = 4\text{V}$

$$\therefore \quad \text{Generated e.m.f} = 250 + 4 = 254\text{V}$$

(b) Armature Cu loss $= I_a^2R_a = 200^2 \times 0.02 = 800\text{W}$

Shunt Cu loss $= VI_{sh} = 250 \times 5 = 1{,}250\text{W}$

$\therefore$ Total Cu loss $= 1{,}250 + 800 = 2{,}050\text{W}$

(c) Stray losses $= 950\text{W}$

Total losses = Total Cu loss + Stray losses

Total losses = 2,050 + 950 = 3,000 W

Output $= 250 \times 195 = 48{,}750\text{W}$
Input $= 48{,}750 + 3{,}000 = 51{,}750\text{W}$
$\therefore$ Output of prime mover $= 51{,}750\text{W}$
(d) Generator input $= 51{,}750\text{W}$; Stray losses $= 950\text{W}$
Electrical power produced in armature $= 51{,}750 - 950 = 50{,}800\text{W}$

$$\eta_m = \left(\frac{50{,}800}{51{,}750}\right) \times 100 = 98.16\%$$

Electrical or Cu losses $= 2{,}050\text{W}$

$$\therefore \quad \eta_e = \frac{48{,}750}{48{,}750+2{,}050} \times 100 = 95.96\%$$

$$\eta_c = \left(\frac{48{,}750}{51{,}750}\right) \times 100 = 94.2\%$$

Example 8: A long shunt dynamo running at 1,000 $r.p.m$ supplies 20kW at a terminal voltage of 220V. The resistance of armature, shunt field, and series field are 0.04Ω , 110Ω and 0.05Ω respectively. Overall efficiency at the above load is 85%. Find: (*i*) Copper loss, (*ii*) Iron and friction loss.

Solution:

Load current $(I_L) = \frac{20{,}000}{220} = 90.91\text{A}$; Shunt field current $(I_f) = \frac{220}{110} = 2\text{A}$

Armature current, $I_a = I_L + I_f = 90.91 + 2 = 92.91\text{A}$

$$\eta_c = \frac{\text{generator output power}}{\text{generator input power}}$$

$$\therefore \quad \text{generator input power} = \frac{\text{generator output power}}{\eta_c} = \frac{20{,}000}{0.85} = 23{,}529\text{W}$$

Total losses in the machine
$= \text{Input power} - \text{Output power} = 23{,}529 - 20{,}000 = 3{,}529\text{W}$

(*i*) Copper loss:
Power loss in series field winding + armature winding $= 92.91^2 \times 0.09 = 777\text{W}$
Power loss in shunt field circuit $= 2^2 \times 110 = 440\text{W}$
Total copper losses $= 777 + 440 = 1{,}217\text{W}$
(*ii*) Iron and friction loss = Total losses − Copper losses
$= 3{,}529 - 1{,}217 = 2{,}312\text{W}$

Example 9: A 4-pole D.C generator is delivering 20A to a load of 10Ω. If the armature resistance is 0.5Ω and the shunt field resistance is 50Ω, calculate the induced e.m.f. and the efficiency of the machine. Allow a drop of 1V per brush.

Solution:

Terminal voltage $= 20 \times 10 = 200\text{V}$

$$I_{sh} = \frac{200}{50} = 4\text{A} \quad ; \quad I_a = 20 + 4 = 24\text{A}$$

$I_a R_a = 24 \times 0.5 = 12V$
Brush drop $= 2 \times 1 = 2V$
$\therefore \quad E_g = 200 + 12 + 2 = 214W$, as in Figure (5)
Since iron and friction losses are not given, only electrical efficiency of the machine can be found out.
Total power generated in the armature
$= 214 \times 24 = 5{,}136W$
Useful output $= 200 \times 20 = 4{,}000W$
$\therefore \quad \eta_e = \frac{4{,}000}{5{,}136} = 0.778$ or 77.88%

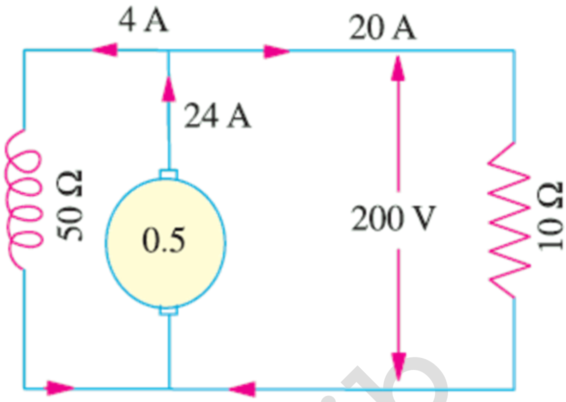

Figure (5)

Example 10: A shunt generator supplies 96A at terminal voltage of 200V. The armature and shunt field resistances are 0.1Ω and 50Ω respectively. The iron and frictional losses are 2,500W Find: (i) e.m.f. generated (ii) copper losses (iii) commercial efficiency.
Solution:
Figure (6) shows the connections of shunt generator.
(i) $I_{sh} = \frac{200}{50} = 4A$
$I_a = I_L + I_{sh} = 94 + 6 = 100A$
$E_g = V + I_a R_a = 200 + (100 \times 0.1) = 210V$
(ii) Armature Cu loss
$= I_a^2 R_a = (100)^2 \times 0.1 = 1{,}000W$
Shunt Cu loss
$= I_{sh}^2 R_{sh} = (4)^2 \times 50 = 800W$
Total Cu loss $= 1{,}000 + 800 = 1{,}800W$
(iii) Total losses = Rotational losses + Cu losses $= 2{,}500 + 1{,}800 = 4{,}300W$
Output Power $= 96 \times 200 = 19{,}200W$
Input power $= 19{,}200 + 4{,}300 = 23{,}500W$
$\therefore \quad \eta_c = \frac{19{,}200}{23{,}500} \times 100 = 81.7\%$

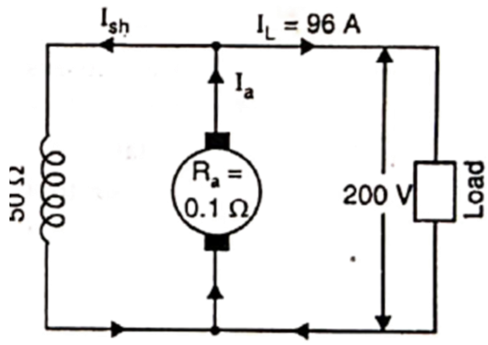

Figure (6)

Example 11: A 75kW shunt generator is operated at 230V. The rotational losses are 1,810W and shunt field circuit draws 5.35A. The armature circuit has a resistance of 0.035Ω and brush drop is 2.2V. Calculate (i) total losses (ii) output of prime mover (iii) efficiency at rated load.

Solution:

(*i*) Load current, $I_L = \frac{75\times10^3}{230} = 326.08\text{A}$

Armature current, $I_a = I_L + I_{sh} = 326.08 + 5.35 = 331.43\text{A}$

Armature Cu loss $I_a^2 R_a = (331.43)^2 \times 0.035 \cong 3{,}845\text{W}$

Shunt Cu loss $= VI_{sh} = 230 \times 5.35 = 1{,}230.5\text{W}$

Brush power loss $=$ Brush drop $\times I_a = 2.2 \times 331.43 \cong 729\text{W}$

Total losses $= 1{,}810 + 3{,}845 + 1{,}230.5 + 729 \cong 7{,}615\text{W}$

(*ii*) Output power $= 75\text{kW} = 75{,}000\text{W}$

Output of prime mover $= 75{,}000 + 7{,}615 = 82{,}615\text{W}$

(*iii*) $\eta = \frac{75{,}000}{82{,}615} \times 100 = 90.78\%$