UNIVERSITY OF DIYALA

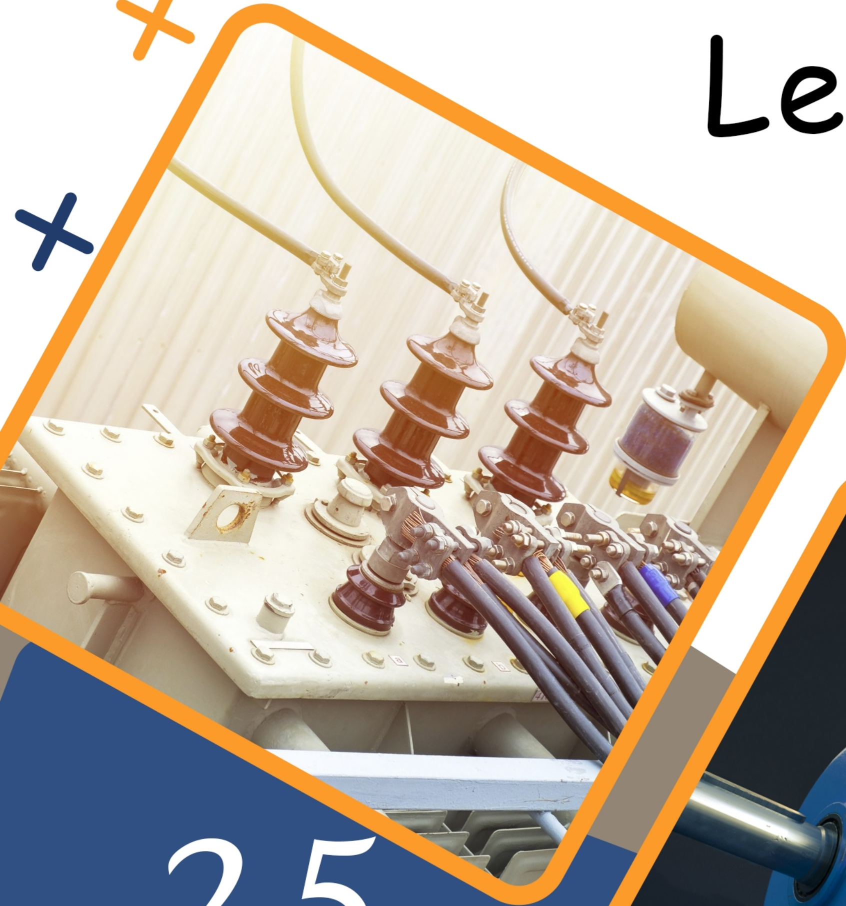
COLLEGE OF ENGINEERING

DEPARTMENT OF THE ELECTRICAL POWER AND MACHINES ENGINEERING

Second Stage

ELECTRICAL MACHINES

Lecture: 5



Lecturer

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Torque Equation of a D.C. Motor:

Torque (T) is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts. Consider a pulley of radius r meter acted upon by a circumferential force of F Newton which causes it to rotate at N in $r.p.m.$ Figure (1).

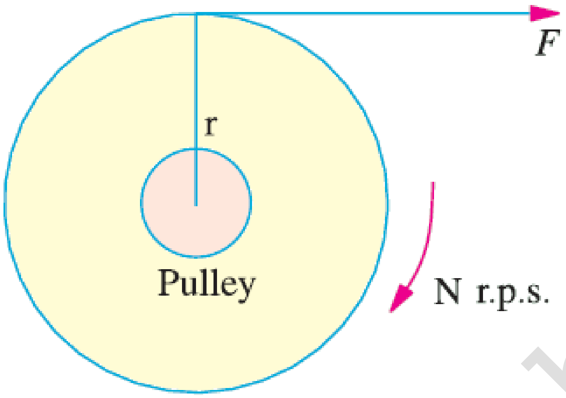

Figure (1)

$T = F \times r$ Newton-meter (N. m)

Work done by this force in one revolution = Force × distance = $F \times 2\pi r$ Joule

Power developed = $F \times 2\pi r \times N$ in Joule/second or Watt

Power developed = $(F \times r) \times 2\pi N$ Watt

Power developed $= T \times \omega$ Watt

Where $\omega = 2\pi N$, ω = angular velocity in radian/second

Moreover, if N is in $r.p.m.$,

Then $\omega = 2\pi N/60$ rad/s

$\therefore \quad P = \frac{2\pi N}{60} \times T = \frac{NT}{9.55}$

Armature Torque (T_a) of a Motor:

If (T_a) is in N. m of a motor running at (N)in $r.p.s.$ then,

power developed $= T_a\omega = T_a \times (2\pi N)$ Watt (*i*)

We also know that electrical power converted into mechanical power in the armature is, $= E_b I_a$ Watt (*ii*)

Equating (*i*) and (*ii*), we get $T_a \times (2\pi N) = E_b I_a$ (*iii*)

Since $E_b = \Phi Z N \frac{P}{A}$ volt, we have

$$T_a \times 2\pi N = \Phi Z N \frac{P}{A} I_a$$

or $T_a = \frac{1}{2\pi} \Phi Z I_a \left(\frac{P}{A}\right)$ N. m

$\therefore \quad T_a = 0.159\Phi Z I_a\left(\frac{P}{A}\right)$ N.m

Note: From the above equation for the torque, we find that $T_a \propto \Phi I_a$.

A. In the case of a series motor, Φ is directly proportional to I_a (before saturation) because field windings carry full armature current $\therefore\ T_a \propto I_a^2$

B. For shunt motors, Φ is practically constant, hence $T_a \propto I_a$.

As seen from (iii) above, $T_a = \frac{E_b I_a}{2\pi N/60}$ N.m (N in $r.p.s.$)

If N is in $r.p.m.$, then, $T_a = \frac{E_b I_a}{2\pi N/60} = \frac{60}{2\pi}\frac{E_b I_a}{N} = 9.55\frac{E_b I_a}{N} = 9.55\frac{P_m}{N}$ N.m

Shaft Torque (T_{sh}):

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as shaft torque T_{sh}. It is so called because it is available at the shaft. The motor output is given by

Output power, $P_{out} = T_{sh} \times 2\pi N$ Watt

Where, (T_{sh} is in N.m and N in $r.p.s.$)

$\therefore \quad T_{sh} = \frac{P_{out}}{\omega} = \frac{\text{Output in watts } (P_{out})}{2\pi N}$ N.m ; Where, (P_{out} in watt and N in $r.p.s.$)

$T_{sh} = \frac{\text{Output in watts } (P_{out})}{2\pi N/60}$ N.m ; Where, (P_{out} in watt and N in $r.p.m.$)

$T_{sh} = \frac{60}{2\pi}\frac{\text{Output in watts } (P_{out})}{N} = 9.55\frac{P_{out}}{N}$ N.m

The difference ($T_a - T_{sh}$) is known as lost torque T_f and is due to iron and friction losses of the motor.

$T_a = T_f + T_{sh}$ Net output of motor $= P_{out} = T_{sh}\omega$

Useful output power $= \frac{2\pi N\, T_{sh}}{60\times 746}$ $h.p.$

or $b.h.p. = \frac{2\pi N\, T_{sh}}{60\times 746}$

Note:

$T_a = \frac{P_m}{\omega}$; $T_{sh} = \frac{P_{out}}{\omega}$; $T_f = \frac{W_{stray}}{\omega}$, Where, ($\omega = 2\pi N$) & N in $r.p.m.$

No Load Condition of a Motor:

On no load, the load requirement is absent, so $T_{sh} = 0$.
So on no load, motor keeps on rotating at a speed of $N_0\, r.p.m.$ drawing an armature current of I_{a0}. This is just enough to produce a torque T_{a0} which satisfies the friction, windage and iron losses of the motor. On no load, speed of the motor is large hence E_b is also large hence $(V - E_{bo})$ is very small hence armature current I_{a0} is also small. So, motor draws less current on no load and takes more and more current as motor load increases.

Torque and Speed Equations:

Before analyzing the various characteristics of motors, let us revise the torque and speed equations as applied to various types of motors.

$T \propto \Phi I_a$ from torque equation.

This is because $0.159\frac{PZ}{A}$ is a constant for a given motor.

Now Φ is the flux produced by the field winding and is proportional to the current passing through the field winding.

$\Phi \propto I_{field}$

But for various types of motors, current through the field winding is different. Accordingly torque equation must be modified.
For a D.C. shunt motor, I_{sh} is constant as long as supply voltage is constant. Hence Φ flux is also constant.

$T \propto I_a$ for shunt motor.

For a D.C. series motor, I_{se} is same as I_a. Hence flux Φ is proportional to the armature current I_a.

$T \propto I_a \Phi \propto I_a^2$ for series motor.

Similarly, as $E_b = \frac{\Phi Z N}{60}\frac{P}{A}$, we can write the speed equation as,

$E_b \propto \Phi N$ $N \propto E_b/\Phi$

Speed of a D.C. Motor:

From the voltage equation of a motor, we get

$$E_b = V - I_a R_a \quad \text{or} \quad \frac{\Phi Z N}{60}\left(\frac{P}{A}\right) = V - I_a R_a$$

$$\therefore \quad N = \frac{V - I_a R_a}{\Phi} \times \left(\frac{60A}{ZP}\right)\ r.p.m.$$

Now, $V - I_a R_a = E_b \quad \therefore \quad N = \frac{E_b}{\Phi} \times \left(\frac{60A}{ZP}\right)\ r.p.m.$ or $N = K\frac{E_b}{\Phi}$

It shows that speed is directly proportional to back e.m.f. E_b and inversely to the flux Φ on $N \propto \frac{E_b}{\Phi}$.

For Series Motor

Let: N_1 = Speed in the 1st case ; I_{a1} = armature current in the 1st case

Φ_1 = flux/pole in the first case

N_2, I_{a2}, Φ_2 = corresponding quantities in the 2nd case

Then, using the above relation, we get

$$N_1 \propto \frac{E_{b1}}{\Phi_1} \text{ where } E_{b1} = V - I_{a1}R_a \quad ; \quad N_2 \propto \frac{E_{b2}}{\Phi_2} \text{ where } E_{b2} = V - I_{a2}R_a$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Prior to saturation of magnetic poles ;

$$\Phi \propto I_a \quad \therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

For Shunt Motor

In this case the same equation applies,

$$i.e., \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

If $\Phi_2 = \Phi_1$, then $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$.

But $V = E_b + I_a R_a$ neglecting brush drop

$$E_b = V - I_a R_a$$

$\therefore$ Speed equation becomes,

$$N \propto \frac{V - I_a R_a}{\Phi}$$

So for shunt motor as flux Φ is constant,

$$\therefore \quad N \propto V - I_a R_a$$

While for series motor, flux Φ is proportional to I_a,

$$\therefore \quad N \propto \frac{V - I_a R_a - I_a R_{se}}{I_a}$$

These relations play an important role in understanding the various characteristics of different types of motors.

Speed Regulation:

The speed regulation for a D.C. motor is defined as the ratio of change in speed corresponding to no load and full load condition to speed corresponding to full load.

Mathematically it is expressed as.

$$\%\text{speed regulation} = \frac{N_{no\ load} - N_{full\ load}}{N_{full\ load}} \times 100$$

Example 1: A D.C motor takes an armature current of 110A at 480V. The armature circuit resistance is 0.2Ω. The machine has 6-poles and the armature is lap-connected with 864 conductors. The flux per pole is 0.05Wb. Calculate *(i)* the speed, *(ii)* the gross torque developed by the armature.

Solution:

$$I_a = 110\text{A}$$

$$E_b = V - I_a R_a = 480 - (110 \times 0.2) = 458\text{V}, \quad \Phi = 0.05\text{Wb}, \quad Z = 864$$

Now, $E_b = \frac{\Phi Z N}{60}\left(\frac{P}{A}\right) \Rightarrow\Rightarrow 458 = \frac{0.05 \times 864 \times N}{60} \times \left(\frac{6}{6}\right) \Rightarrow\Rightarrow N \cong 636\ r.p.m.$

$$T_a = 0.159 \Phi Z I_a \frac{P}{A} = 0.159 \times 0.05 \times 864 \times 110 \left(\frac{6}{6}\right) \cong 755.6\ \text{N.m}$$

Or, $T_a = \frac{P_m}{\omega} = \frac{E_b I_a}{2\pi N/60} = 9.55 \frac{E_b I_a}{N} = 9.55 \frac{458 \times 110}{636} \cong 756.5\ \text{N.m}$

Example 2: A 250V, 4-pole, wave-wound D.C series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75Ω. The motor takes a current of 40A. Estimate its speed and gross torque developed if it has a flux per pole of 25mWb.

Solution:

$$E_b = V - I_a R_a = 250 - (40 \times 0.75) = 220\text{V}$$

$$\therefore \quad E_b = \frac{\Phi Z N}{60}\left(\frac{P}{A}\right) \Rightarrow\Rightarrow 220 = \frac{25 \times 10^{-3} \times 782 \times N}{60} \times \frac{4}{2} \Rightarrow\Rightarrow N \cong 337.6\ r.p.m.$$

$$T_a = 0.159 \Phi Z I_a \left(\frac{P}{A}\right) = 0.159 \times 25 \times 10^{-3} \times 782 \times 40 \times \left(\frac{4}{2}\right) \cong 249\ \text{N.m}$$

Example 3: Determine developed torque and shaft torque of 220V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2kW by taking 45A from the mains. The flux per pole is 25mWb and its armature circuit resistance is 0.6Ω.

Solution:

Developed torque or gross torque is the same thing as armature torque.

$$\therefore \quad T_a = 0.159 \Phi Z I_a \left(\frac{P}{A}\right) = 0.159 \times 25 \times 10^{-3} \times 800 \times 45 \times \left(\frac{4}{2}\right) = 286.2\ \text{N.m}$$

$$E_b = V - I_a(R_a + R_{se}) = 220 - (45 \times [0.6 + 0]) = 193\text{V}$$

Note: R_{se} is negligible because its value is small and not given.

Now, $E_b = \Phi Z N \left(\frac{P}{A}\right) \Rightarrow\Rightarrow 193 = 25 \times 10^{-3} \times 800 \times N \times \left(\frac{4}{2}\right)$

$$\therefore \quad N = 4.825\ r.p.s.$$

Also, $2\pi N T_{sh} = \text{output} \Rightarrow\Rightarrow 2\pi \times 4.825 \times T_{sh} = 8{,}200 \Rightarrow\Rightarrow T_{sh} = \frac{P_{out}}{2\pi N}$

$$\therefore \quad T_{sh} \cong 270.5\ \text{N.m}$$

Example 4: A 220V D.C. shunt motor runs at 500 $r.p.m.$ when the armature current is 50A. Calculate the speed if the torque is doubled. Given that $R_a = 0.2\Omega$.

Solution:

$T_a \propto \Phi I_a$. Since Φ is constant in shunt motor, $T_a \propto I_a$

$\therefore T_{a1} \propto I_{a1}$ ...(1)

$T_{a2} \propto I_{a2}$ ...(2)

Divide equation (2) on equation (1), we get;

$T_{a2}/T_{a1} = I_{a2}/I_{a1}$

As torque is doubled, $T_{a2} = 2T_{a1}$ $\quad T_{a2}/T_{a1} = 2$

$\therefore \quad 2 = \frac{I_{a2}}{50} \Rightarrow\Rightarrow I_{a2} = 100\text{A}$

$E_{b1} = V - I_{a1}R_a = 220 - (50 \times 0.2) = 210\text{V},$

$E_{b2} = V - I_{a2}R_a = 220 - (100 \times 0.2) = 200\text{V}$

$E_b = \frac{\Phi ZN}{60}\frac{P}{A} \Rightarrow\Rightarrow E_b \propto N$, as $\left(\frac{\Phi ZN}{60}\frac{P}{A}\right)$ is constant.

$E_{b1} \propto N_1$ ...(3)

$E_{b2} \propto N_2$ ...(4)

Divide equation (4) on equation (3), we get;

Now, $E_{b2}/E_{b1} = N_2/N_1$

since Φ remains constant for shunt motor.

$\therefore \quad \frac{200}{210} = \frac{N_2}{500} \Rightarrow\Rightarrow N_2 = 476.19\ r.p.m.$

Example 5: A 500V, 37.3kW, 1,000 $r.p.m.$ D.C. shunt motor has on full-load an efficiency of 90 percent. Determine (i) full-load line current (ii) full load shaft torque in N. m.

Solution:

(i) $P_{out} = 37.3\text{kW} = 37{,}300\text{W}$; Motor input $= \frac{37{,}300}{0.9} \cong 41{,}444\text{W}$

F.L. line current $= \frac{41{,}444}{500} \cong 82.9\text{A}$

(ii) $T_{sh} = 9.55\frac{P_{out}}{N} = 9.55 \times \frac{37{,}300}{1{,}000} = 356.215\ \text{N.m}$

Example 6: A 4-pole, 220V shunt motor has 540 lap-wound conductor. It takes 32A from the supply mains and develops output power of 5,595kW. The field winding takes 1A. The armature resistance is 0.09Ω and the flux per pole is 30mWb. Calculate (i) the speed and (ii) the torque developed in newton-meter.

Solution:

$I_a = I_L - I_{sh} = 32 - 1 = 31\text{A}$

$E_b = V - I_aR_a = 220 - (0.09 \times 31) = 217.21\text{V}$

Now, $E_b = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \Rightarrow\Rightarrow 217.21 = \frac{30\times10^{-3}\times540\times N}{60}\times\left(\frac{4}{4}\right)$

(*i*) ∴ $N = 804.48\ r.p.m.$

(*ii*) $T_{sh} = 9.55\times\frac{P_{\text{out}}\text{ in watts}}{N} = 9.55\times\frac{5{,}595}{804.48} \cong 66.42\ \text{N.m}$

Example 7: Find the No-load and full-load speeds for a four-pole, 220V, and 20kW, shunt motor having the following data:
Field current = 5A ; armature resistance = 0.04Ω ; Flux per pole = 0.04Wb ;
Number of armature-conductors = 160 ; Two-circuit wave-connection ;
Full load current = 95A ; No load current = 9A. Neglect armature reaction.

Solution:

For shunt motor, I_{sh} is constant for both cases.

At No Load:

$I_{\text{no-load}} = 9\text{A}\ ;\ I_{sh} = 5\text{A} \Rightarrow\Rightarrow I_a = I_L - I_{sh} = 9 - 5 = 4\text{A}$

$E_{b0} = V - I_a R_a = 220 - (4\times0.04) = 219.84\text{V}$

$E_b = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \Rightarrow\Rightarrow 219.84 = \frac{0.04\times160\times N_0}{60}\left(\frac{4}{2}\right)$

∴ No-Load speed, $N_0 = 1{,}030.5\ r.p.m.$

At Full-Load:

$I_{\text{full-load}} = 95\text{A}\ ;\ I_{sh} = 5\text{A} \Rightarrow\Rightarrow I_a = I_L - I_{sh} = 95 - 5 = 90\text{A}$

$E_b = V - I_a R_a = 220 - (90\times0.04) = 216.4\text{V}$

$E_b = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \Rightarrow\Rightarrow 216.4 = \frac{0.04\times160\times N_{\text{full-load}}}{60}\frac{4}{2}$

∴ Full-load speed, $N_{\text{full-load}} \cong 1{,}014.4\ r.p.m.$

Or, $\frac{E_b}{E_{b0}} = \frac{N_{\text{full-load}}}{N_0} \Rightarrow N_{\text{full-load}} = N_0\frac{E_b}{E_{b0}} = 1{,}030.5\times\frac{216.4}{219.84} \cong 1{,}014.4\ r.p.m.$

Example 8: A D.C. series motor takes 40A at 220V and runs at 800 $r.p.m.$ If the armature and field resistance are 0.2Ω and 0.1Ω respectively and the iron and friction losses are 0.5kW, find the torque developed in the armature. What will be the output of the motor?

Solution:

$E_b = V - I_a(R_a + R_{se}) = 220 - (40\times[0.2+0.1]) = 208\text{V}$

$T_a = 9.55\frac{E_b I_a}{N} = 9.55\times\frac{208\times40}{800} = 99.32\ \text{N.m}$

Cu loss in armature and series-field resistance = $40^2\times0.3 = 480\text{W}$

Iron and friction losses = 500W

∴ Total losses = 480 + 500 = 980W

Motor power input = $VI_L = 220\times40 = 8{,}800\text{W}$

Motor output, $P_{out} = P_{in}$ − Total losses = 8,800 − 980 = 7,820W = 7.82kW

Example 9: A 250V shunt motor runs at 1,000 $r.p.m.$ at no-load and takes 8A. The total armature and shunt field resistances are respectively 0.2Ω and 250Ω. Calculate the speed when loaded and taking 50A. Assume the flux to be constant.

Solution:

Formula used: $\frac{N}{N_0} = \frac{E_b}{E_{b2}} \times \frac{\Phi_0}{\Phi}$

Since $\Phi_0 = \Phi$ (given)

$$\frac{N}{N_0} = \frac{E_b}{E_{b0}}$$

$$I_{sh} = \frac{250}{250} = 1\text{A}$$

$$E_{b0} = V - I_{a0}R_a = 250 - (7 \times 0.2) = 248.6\text{V}$$

$$E_b = V - I_aR_a = 250 - (49 \times 0.2) = 240.2\text{V}$$

$$\therefore \frac{N}{1{,}000} = \frac{240.2}{248.6} \Rightarrow\Rightarrow N = 966.21\ r.p.m.$$

Example 10: A 250V shunt motor runs at 1,000 $r.p.m.$ at no-load and takes 8A. The total armature and shunt field resistances are respectively 0.2Ω and 250Ω. Calculate the speed when loaded and taking 50A. Assume the flux to be constant.

Solution:

$$I_{sh} = \frac{250}{250} = 1\text{A}$$

$$E_{b0} = V - I_{a0}R_a = 250 - (8 \times 0.2) = 248.4\text{V}$$

$$E_b = V - I_aR_a = 250 - (50 \times 0.2) = 240\text{V}$$

$$\frac{E_b}{E_{b0}} = \frac{N}{N_0} \Rightarrow\Rightarrow \frac{240}{248.4} = \frac{N}{1{,}000} \Rightarrow\Rightarrow N \cong 966.2\ r.p.m.$$

Example 11: A D.C. series motor operates at 800 $r.p.m.$ with a line current of 100A from 230V mains. Its armature circuit resistance is 0.15Ω and its field resistance 0.1Ω. Find the speed at which the motor runs at a line current of 25A, assuming that the flux at this current is 45 percent of the flux at 100A.

Solution:

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}, \quad \Phi_2 = 0.45\Phi_1$$

$$E_{b1} = 230 - ([0.15 + 0.1] \times 100) = 205\text{V}$$

$$E_{b2} = 230 - ([0.15 + 0.1] \times 25) = 223.75\text{V}$$

$$\frac{N_2}{800} = \frac{223.75}{205} \times \frac{\Phi_1}{0.45\Phi_1} \Rightarrow\Rightarrow N_2 \cong 1{,}940.38\ r.p.m.$$

Losses and Efficiency:

The losses taking place in the motor are the same as in generators. These are:
(*i*) Copper losses
(*ii*) Magnetic losses
(*iii*) Mechanical losses.

The condition for maximum *power* developed by the motor is, $I_a R_a = \frac{V}{2} = E_b$

The condition for maximum *efficiency* is that,
armature Cu losses are equal to constant losses.

Power Stages:

The various stages of energy transformation in a motor and also the various losses occurring in it are shown in the flow diagram of Figure (2).

- Overall or commercial efficiency $\eta_c = \frac{C}{A}$
- Electrical efficiency $\eta_e = \frac{B}{A}$
- Mechanical efficiency $\eta_m = \frac{C}{B}$

The efficiency curve for a motor is similar in shape to that for a generator.

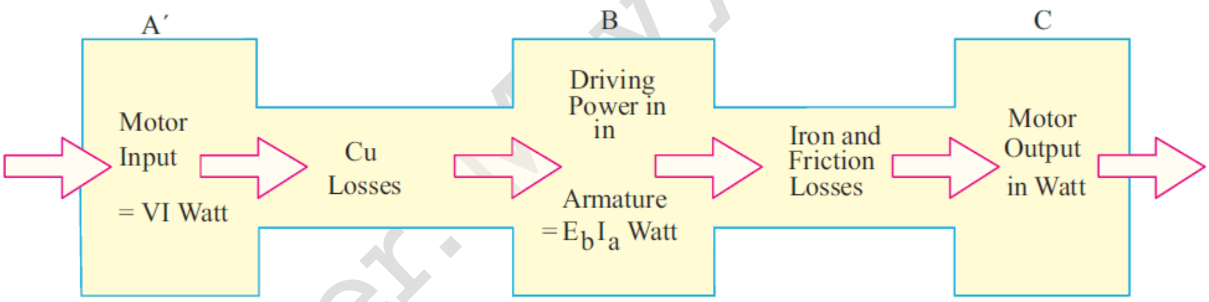

Figure (2): Flow Diagram of Power Stage

It is seen that $A - B$ = copper losses and $B - C$ = iron and friction losses.

Figure (3) shows a comparison between the Flow Diagram of Power Stage for both D.C. generators & D.C. motors.

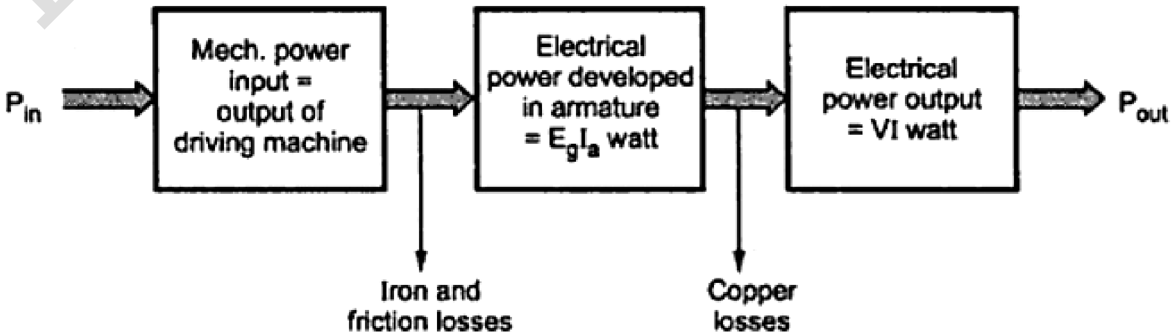

Figure (3-a): Flow Diagram of Power Stage for generators

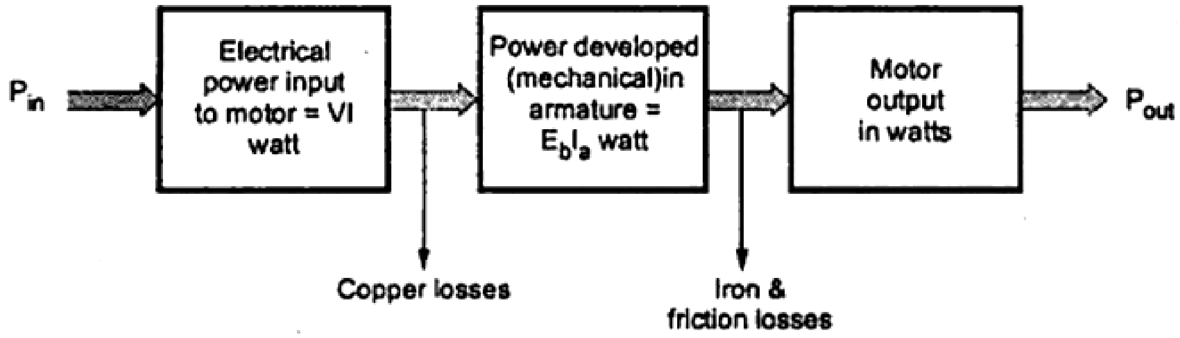

Figure (3-b): Flow Diagram of Power Stage for motors

Example 12: A 220V shunt motor has an armature resistance of 0.2Ω and field resistance of 110Ω. The motor draws 5A at 1,500 $r.p.m.$ at no load. Calculate the speed and shaft torque if the motor draws 52A at rated voltage.

Solution:

When working on no-load:

$$I_{sh} = \frac{220}{110} = 2\text{A} \quad ; \quad I_{a0} = 5 - 2 = 3\text{A}$$

$$E_{b0} = 220 - (3 \times 0.2) = 219.4\text{V}$$

When working on full-load:

$$I_{a1} = 52 - 2 = 50\text{A}$$

$$E_{b1} = 220 - (50 \times 0.2) = 210\text{V}$$

$$\frac{E_{b1}}{E_{b0}} = \frac{N_1}{N_0} \Rightarrow\Rightarrow \frac{210}{219.4} = \frac{N_1}{1{,}500} \Rightarrow\Rightarrow N_1 \cong 1{,}436\, r.p.m.,$$

For shunt motor, $(\Phi_1 = \Phi_0)$

For finding the shaft torque, we will find the motor output when it draws a current of 52A. First we will use the no-load data for finding the constant losses of the motor.

No load motor input $= 220 \times 5 = 1{,}100\text{W}$

No load Arm. Cu loss $= 3^2 \times 0.2 = 1.8\text{W}$

∴ Constant or standing losses of the motor $= 1{,}100 - 1.8 \cong 1{,}098\text{W}$

When loaded, F.L. arm. Cu loss $= 50^2 \times 0.2 = 500\text{W}$

Hence, total motor losses $= 1{,}098 + 500 = 1{,}598\text{W}$

Motor input on load $= 220 \times 52 = 11{,}440\text{W}$

Output $= 11{,}440 - 1{,}598 = 9{,}842\text{W}$

$$\therefore \quad T_{sh} = 9.55 \frac{\text{output}}{N} = 9.55 \times \frac{9{,}842}{1{,}436} = 65.45\ \text{N.m}$$

Example 13: A 250V D.C. shunt motor runs at 1,000 $r.p.m.$ while taking a current of 25A. Calculate the speed when the load current is 50A if armature reaction weakens the field by 3%. Determine torques in both cases.

$R_a = 0.2\Omega$; $R_f = 250\Omega$; Voltage drop per brush is 1V

Solution:

$$I_{sh} = \frac{250}{250} = 1\text{A}$$
$$I_{a1} = 25 - 1 = 24\text{A}$$
$$E_{b1} = 250 - \text{arm. drop} - \text{brush drop}$$
$$E_{b1} = 250 - (24 \times 0.2) - 2 = 243.2\text{V}$$
$$I_{a2} = 50 - 1 = 49\text{A}$$
$$E_{b2} = 250 - (49 \times 0.2) - 2 = 238.2\text{V}$$
$$\frac{N_2}{1{,}000} = \frac{238.2}{243.2} \times \frac{\Phi_1}{0.97\Phi_1}$$
$$N_2 \cong 1{,}010\ r.p.m.$$
$$T_{a1} = 9.55\frac{E_{b1}I_{a1}}{N_1} = 9.55 \times \frac{243.2 \times 24}{1{,}000} = 55.74\ \text{N.m}$$
$$T_{a2} = 9.55\frac{E_{b2}I_{a2}}{N_2} = 9.55 \times \frac{238.2 \times 49}{1{,}010} = 110.36\ \text{N.m}$$

Example 14: The armature winding of a 4-pole, 250V D.C. shunt motor is lap connected There are 120 slots, each slot containing 8 conductors. The flux per pole is 20mWb and current taken by the motor is 25A. The resistance of armature and field circuit are 0.1Ω and 125Ω respectively. If the rotational losses amount to be 810W find, (*i*) gross torque (*ii*) useful torque and (*iii*) efficiency.

Solution:

$$I_{sh} = \frac{250}{125} = 2\text{A} \quad ; \quad I_a = 25 - 2 = 23\text{A}$$
$$E_b = 250 - (23 \times 0.1) = 247.7\text{V} \quad ; \quad Z_{total} = 120 \times 8 = 960$$

Now, $E_b = \frac{\Phi Z N}{60}\frac{P}{A} \Rightarrow\Rightarrow 247.7 = \frac{20\times10^{-3}\times960\times N}{60} \times \frac{4}{4} \Rightarrow\Rightarrow N = 774\ r.p.m.$

(*i*) Gross torque or armature torque, $T_a = 9.55\frac{E_b I_a}{N} = 9.55 \times \frac{247.7\times23}{774} \cong$ 70.3 N.m

(*ii*) Arm Cu loss $= 23^2 \times 0.1 \cong 53\text{W}$; Shunt Cu loss $= 250 \times 2 = 500\text{W}$

Rotational losses = 810W ; Total motor losses $= 810 + 500 + 53 = 1{,}363\text{W}$

Motor input $= 250 \times 25 = 6{,}250\text{W}$

Motor output $= 6{,}250 - 1{,}363 = 4{,}887\text{W}$

$$T_{sh} = 9.55 \times \frac{P_{out}}{N} = 9.55 \times \frac{4{,}887}{774} \cong 60.3\ \text{N.m}$$

(*iii*) Efficiency $= \frac{4{,}887}{6{,}250} = 0.7819$ or 78.19%