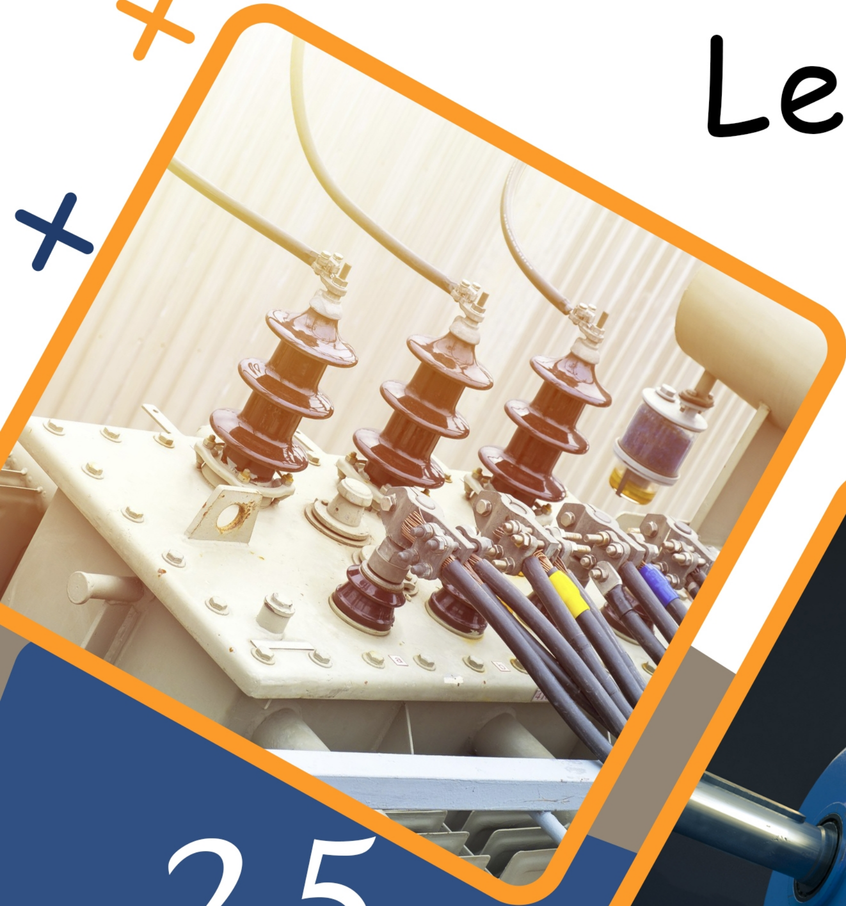
UNIVERSITY OF DIYALA
COLLEGE OF ENGINEERING
DEPARTMENT OF THE ELECTRICAL POWER AND MACHINES ENGINEERING

Second Stage

ELECTRICAL MACHINES

Lecture: 6



Lecturer
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TRANSFORMER

A transformer is a static piece of equipment used either for raising or lowering the voltage of an A.C. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Figure (1). The winding connected to the A.C. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V_1 whose magnitude is to be changed is applied to the primary. Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating e.m.f. E_2 is induced in the secondary. This induced e.m.f. E_2 in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load.

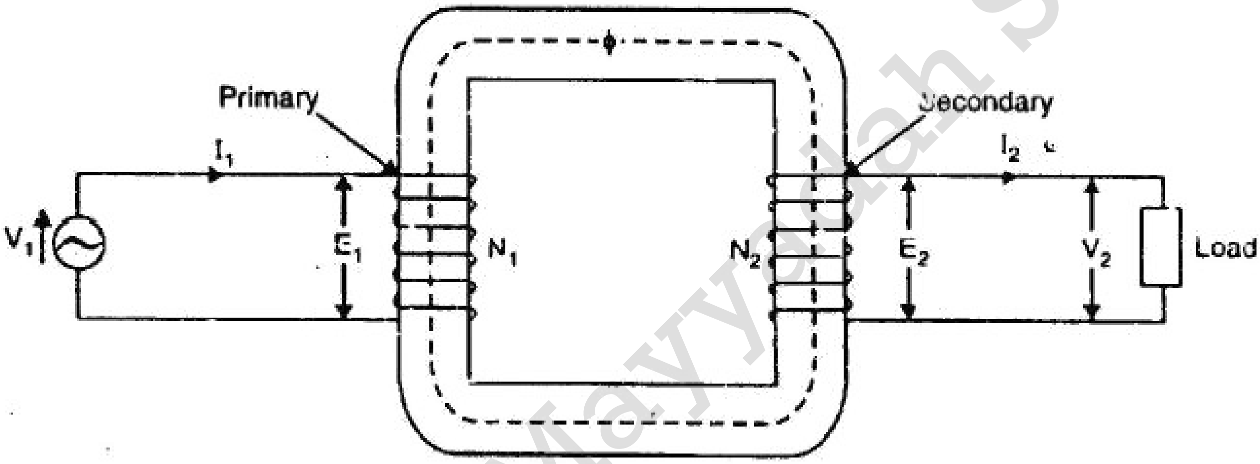


Figure (1)

Types of Transformers on the Basis of Application:

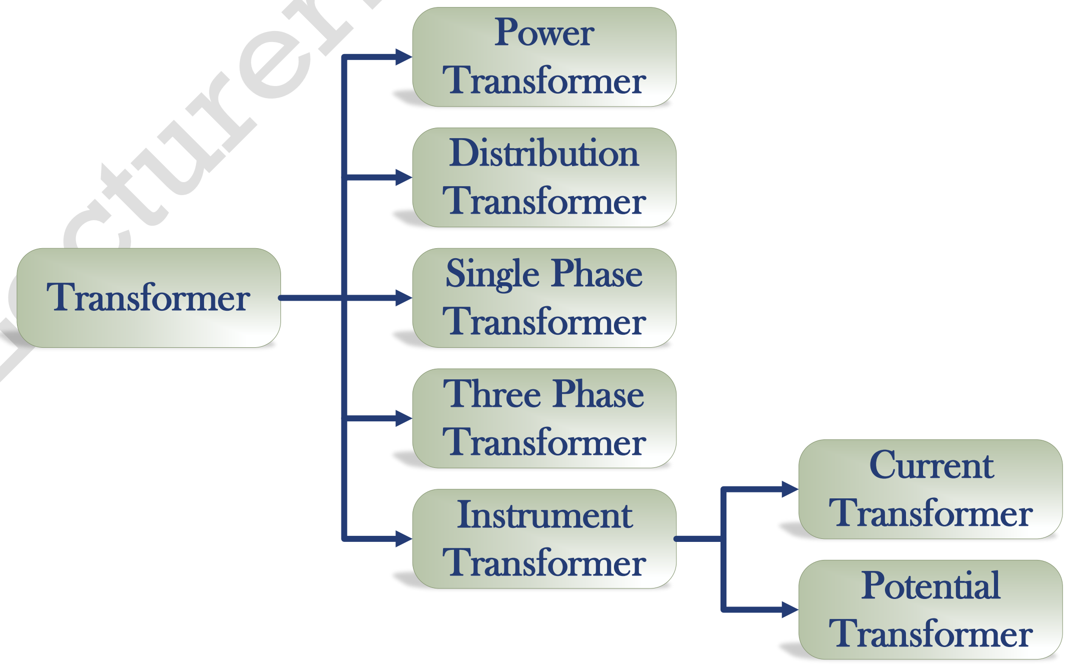


Figure (2)

Power Transformer:

This type of transformer is used for transmission of electric power in high voltage circuit. It is designed for handling higher power. It can either be step-up or step-down type.

Distribution Transformer:

It is used for stepping down the voltage level for the distribution of electric power in the locality. This is generally step-down transformer of voltage rating 11kV/415V. It is pole mounted and often seen in the residential societies.

Single Phase Transformer:

A transformer which receives electrical power from a single-phase source and delivers power to single phase load is called single phase transformer.

Three Phase Transformer:

It is supplied with three phase power source and delivers power to either single phase or three phase loads. The three primary and secondary phases are either connected in STAR or DELTA.

Instrument Transformer:

These transformers are used for the measurement and instrumentation purposes.

- Current Transformer and Potential transformer. Current transformer is used for the measurement of electric current flowing through a circuit.
- Potential Transformer or PT is used for the measurement of voltage of a line.

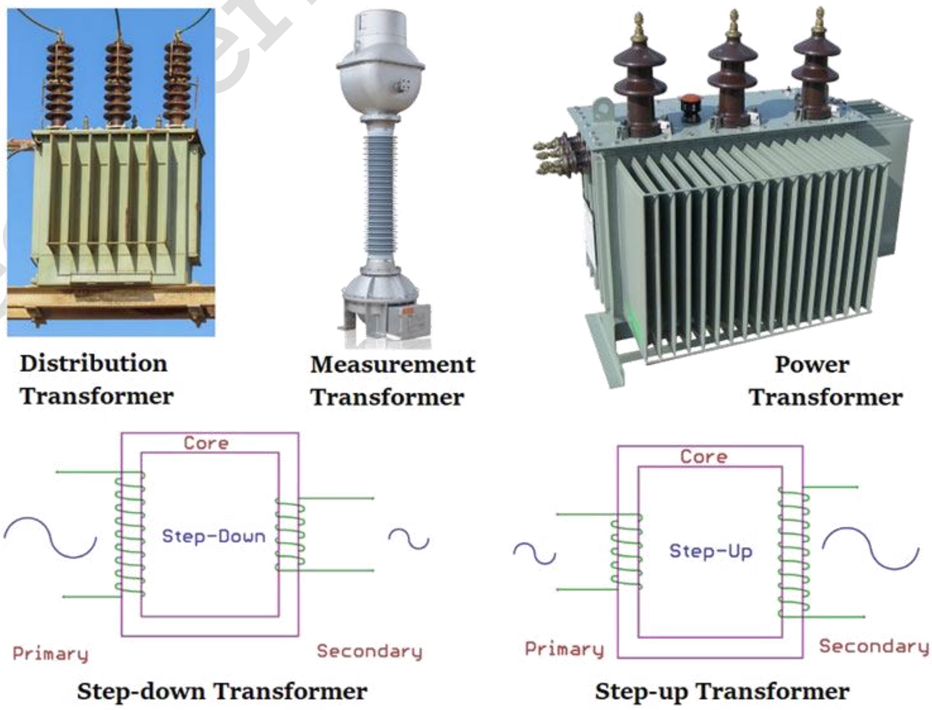


Figure (3): Type of transformer

Principle of Working:

The principle of mutual induction states that when two coils are inductively coupled and if current in one coil is changed uniformly then an e.m.f. gets induced in the other coil. This e.m.f. can drive a current, when a closed path is provided to it. The transformer works on the same principle.

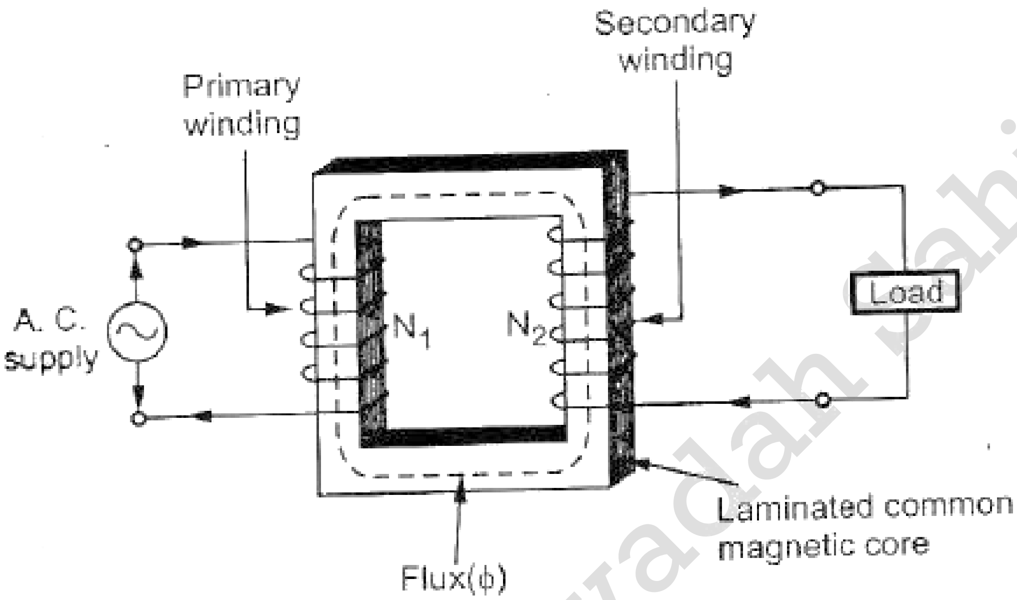


Figure (4)

When primary winding is excited by an alternating voltage, it circulates an alternating current. This current produces an alternating flux (Φ) which completes its path through common magnetic core as shown in Figure (4). thus an alternating flux links with the secondary winding. As the flux is alternating, according to Faraday's law of an electromagnetic induction, mutually induced e.m.f. gets developed in the secondary winding. If now load is connected to the secondary winding, this e.m.f. drives a current through it.

Construction:

There are two basic parts of a transformer
(*i*) Magnetic core (*ii*) Winding or coils.
The core of the transformer is either square or rectangular in size. It is further divided into two parts.
The vertical portion on which coils are wound is called *limb* while the top and bottom horizontal portion is called *yoke of the core*. These parts are shown in the Figure (5-a).
Core is made up of laminations to reduce eddy current losses. Generally high-grade silicon steel laminations [0.3 to 0.5mm thick] are used to reduce hysteresis loss. These laminations are insulated from each other by using insulation like varnish.

Laminations are overlapped so that to avoid the air gap at the joints. For this generally **L** shaped or **T** shaped laminations are used which are shown in the Figure (5-b).

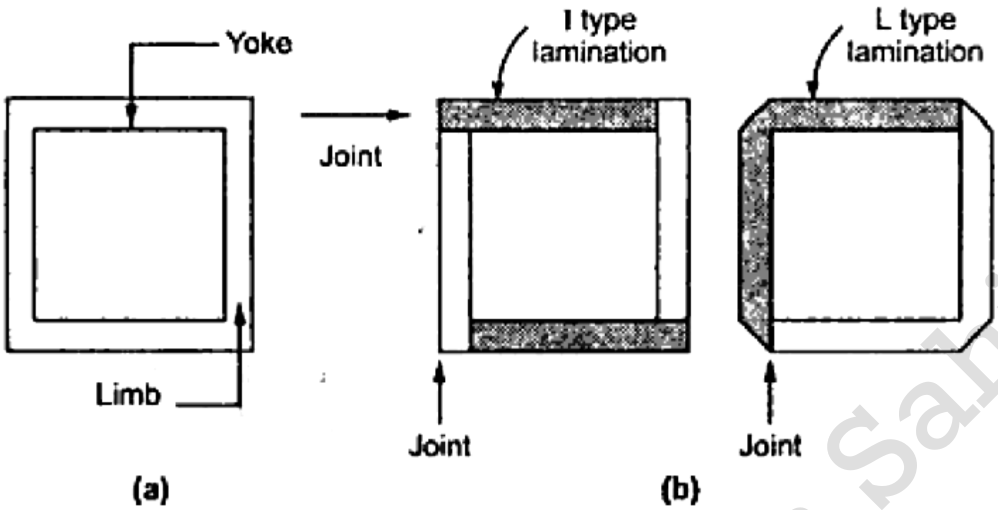


Figure (5): Construction of transformer

The cross-section of the limb depends on the type of coil to be used either circular or rectangular.
The different cross-sections of limbs, practically used are shown in the Figure (6).

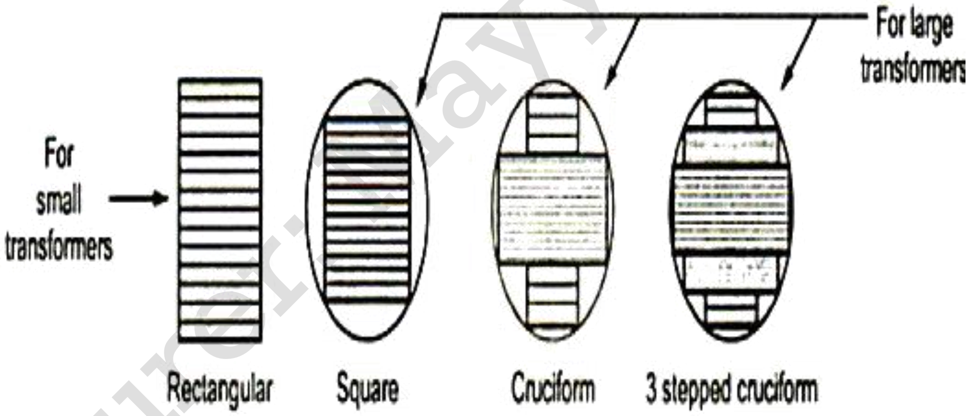


Figure (6): Different cross section

Types of Windings:

The coils used are wound on the limbs and are insulated from each other. In the basic transformer, the two windings wound are shown on two different limbs *i.e.* primary on one limb while secondary on other limb. But due to this leakage flux increases which affects the transformer performance badly. Similarly, it is necessary that the windings should be very closes to each other to have high mutual inductance. To achieve this, the two windings are split into number of coils

and are wound adjacent to each other on the, same limb. A very common arrangement is cylindrical concentric coils as shown in the Figure (7).

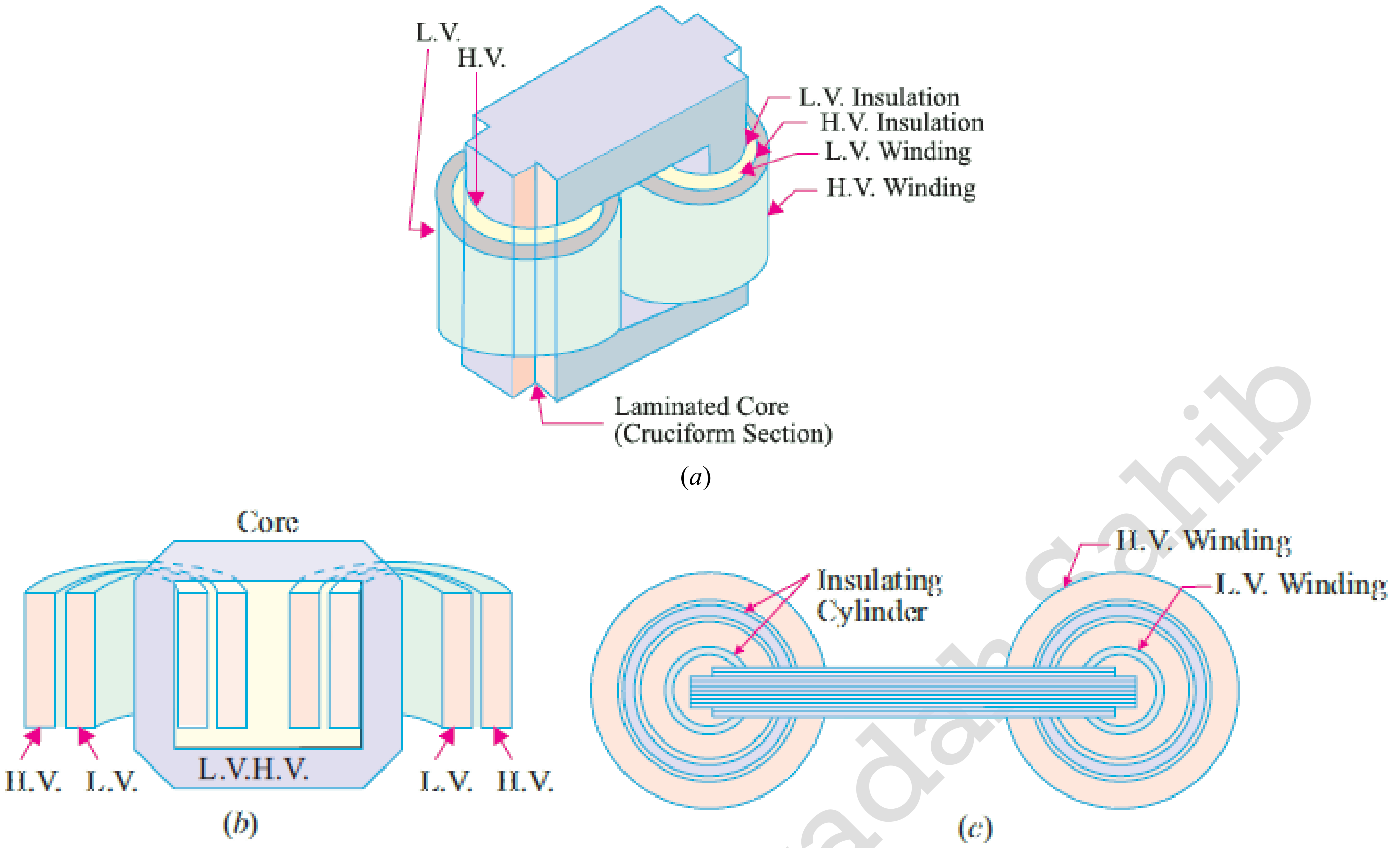


Figure (7): Cylindrical concentric coils

Such cylindrical coils are used in the *core type transformer*. These coils are mechanically strong. These are wound in the helical layers. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core from ease of insulating it from the core. The high voltage is placed after it.

The other type of coils which is very commonly used for the *shell type of transformer* is sandwich coils. Each high voltage portion lies between the two low voltage portion sandwiching the high voltage portion. Such subdivision of windings into small portions reduces the leakage flux. Higher the degree of subdivision, smaller is the reactance. The sandwich coil is shown in the Figure (8). The top and bottom coils are low voltage coils. All the portions are insulated from each other by paper.

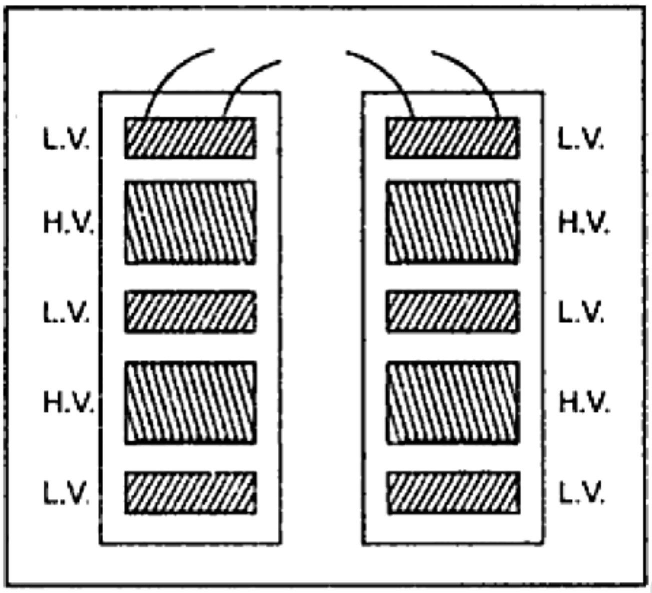


Figure (8): Sandwich coils

Types of Transformers:

The classification of the transformers is based on the relative arrangement or disposition of the core and the windings. There are three main types of the transformers which are:

(*i*) Core type (*ii*) Shell type (*iii*) Berry type

(*i*) Core Type Transformer:

It has a single magnetic circuit. The core is rectangular having two limbs. The winding encircles the core in Figure (9). The coils used are of cylindrical type. Core is made up of large number of thin laminations. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.

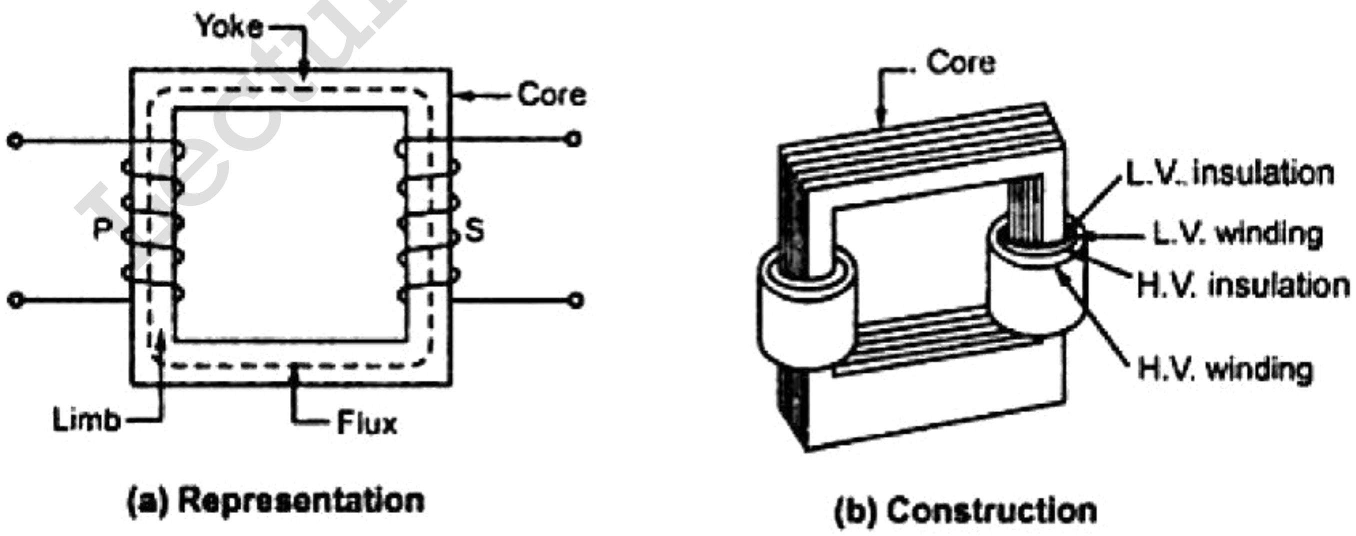


Figure (9): Core type transformer

(*ii*) Shell Type Transformer:

It has a double magnetic circuit. The core has three limbs as shown in Figure (10). Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich coils.

Generally, for very high voltage transformers, the shell type construction is preferred. For removing any winding for maintenance, large number of laminations are required to be removed.

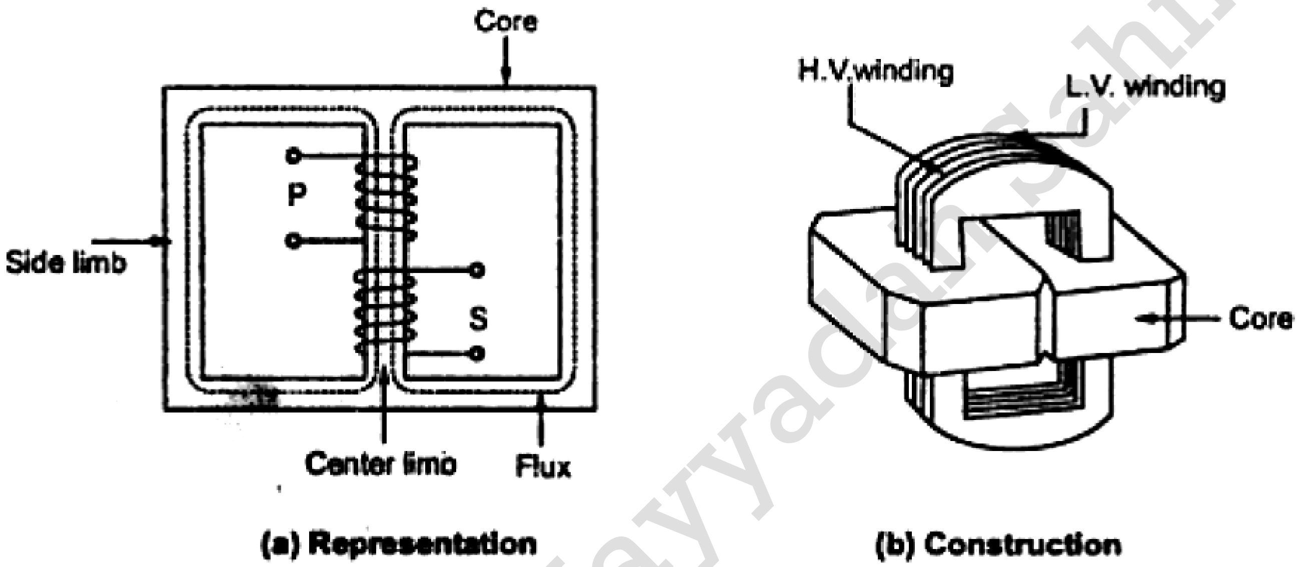


Figure (10): Shell type transformer

(*iii*) Berry Type Transformer:

This has distributed magnetic circuit The number of independent magnetic circuits are more than 2. Its core construction is like spokes of a wheel. Otherwise, it is symmetrical to that of shell type as shown as in the Figure (11).

The transformers are generally kept in tightly fitted sheet metal tanks. The tanks are constructed of specified high quality steel plate. The tanks are filled with the special insulating oil. The entire transformer assembly is immersed in the oil. The oil serves two functions:

(*i*) Keeps the coils cool (*ii*) Provides the transformers an additional insulation

The oil should be absolutely free from alkalies, Sulphur and specially from moisture. Presence of very small moisture lowers the dielectric strength of oil, affecting its performance badly.

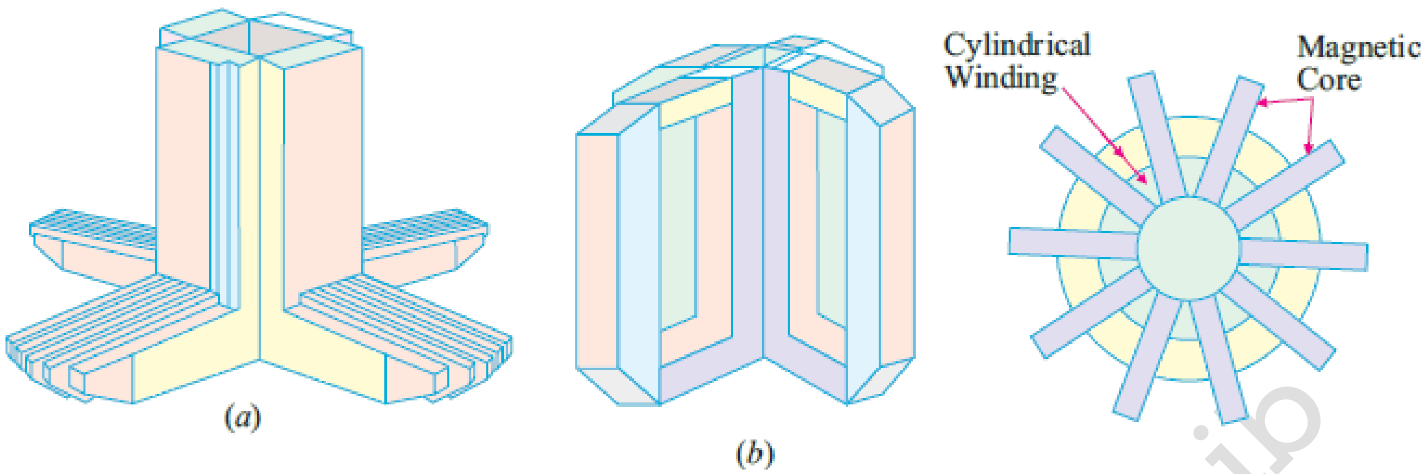


Figure (11): Berry type transformer

No.	Core Type	Shell Type
1	The winding encircles the core.	The core encircles most part of tl windings.
2	It has single magnetic circuit.	It has double magnetic circuits.
3	The core has two limbs.	The core has three limbs.
4	The cylindrical coils are used.	The multilayer disc or sandwich type coi are used.
5	The coils can be easily removed from maintenance point of view.	The coils cannot be removed easily.
6	Preferred for low voltage transformers.	Preferred for high voltage transformers.

Table (1): Comparison between core and shell type transformers

E.M.F. Equation of Transformer:

When the primary winding is excited by an alternating voltage V_t, it circulates alternating current, producing an alternating flux Φ. The primary winding has N_1 number of turns. The alternating flux Φ linking with the primary winding itself induces an e.m.f. in it denoted as E_1. The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f. Let us derive the equations for E_1 and E_2.

The primary winding is excited by purely sinusoidal alternating voltage. Hence the flux produced is also sinusoidal in nature having maximum value of Φ_m as shown in the Figure (12).

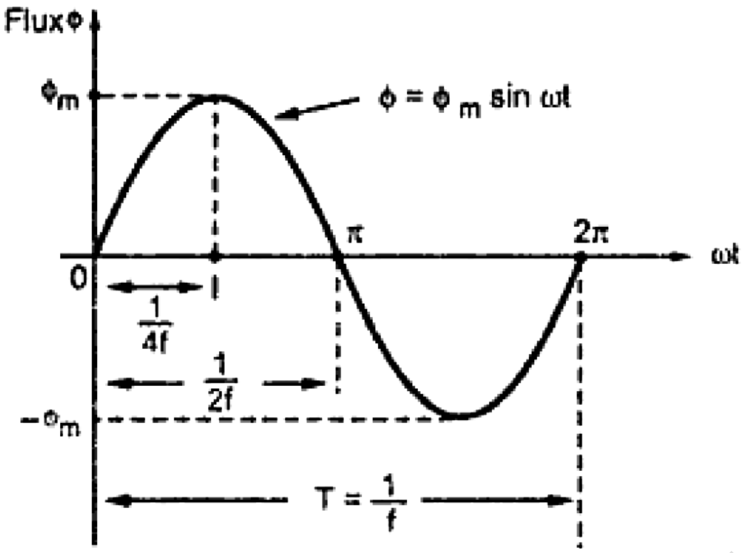


Figure (12): Sinusoidal flux

The various quantities which affect the magnitude of the induced e.m.f. are:
Φ = Flux
Φ_m = Maximum value of flux = $B_m \times A$
N_1 = Number of primary winding turns
N_2 = Number of secondary winding turns
f = Frequency of the supply voltage
E_1 = R.M.S. value of the primary induced e.m.f.
E_2 = R.M.S. value of the secondary induced e.m.f.

From *Faraday's law of electromagnetic induction* the average e.m.f. induced in each turn is proportional to the average rate of change of flux.

$\therefore$ Average e.m.f. per turn = Average rate of change of flux

$\therefore$ Average e.m.f. per turn = $\frac{d\Phi}{dt}$

Now, $\frac{d\Phi}{dt} = \frac{\text{change in flux}}{\text{Time required for change in flux}}$

Consider the $(1/4)^{th}$ cycle of the flax as shown in the Figure (12). Complete cycle gets completed in (1/f) seconds. In $(1/4)^{th}$ time period, the change in flux is from 0 to Φ_m.

$$\frac{d\Phi}{dt} = \frac{\Phi_m - 0}{(1/4f)} \quad \text{as } (dt) \text{ for } (1/4)^{th} \text{ time period is } (1/4f) \text{ seconds}$$

$$= 4f\Phi_m \ \text{Wb/Sec}$$

$\therefore$ Average e.m.f./turn = $4f\Phi_m$ volts

As Φ is sinusoidal, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature. For sinusoidal quantity, Form Factor (F.F.) $= \frac{\text{R.M.S.value}}{\text{Average value}} = 1.11$

$\therefore$ R.M.S. value = 1.11 × Average value

$\therefore$ R.M.S. value of induced e.m.f./turn $= 1.11 \times 4f\Phi_m = 4.44f\Phi_m$

$\therefore$ R.M.S. value of induced e.m.f. for whole primary windings

$E_1 = N_1 \times 4.44f\Phi_m$ volts

$\therefore$ R.M.S. value of induced e.m.f. for whole secondary windings

$E_2 = N_2 \times 4.44f\Phi_m$ volts

The expressions of E_1 & E_2 are called e.m.f. equations of transformer:

$$E_1 = N_1 \times 4.44f\Phi_m \quad \ldots (1)$$

$$E_2 = N_2 \times 4.44f\Phi_m \quad \ldots (2)$$

It is seen from (1) and (2) that $\left(\frac{E_1}{N_1}\right) = \left(\frac{E_2}{N_2}\right) = 4.44f\Phi_m$.

It means that e.m.f./turn is the same in both the primary and secondary windings.

Ratio of Transformer:

(*i*) Voltage ratio:

From the e.m.f. equation of transformer,

$E_1 = N_1 \times 4.44f\Phi_m$ & $E_2 = N_2 \times 4.44f\Phi_m$.

Taking the ratio of the two equations we get,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

This ratio secondary induced e.m.f. to primary induced e.m.f. is known as *voltage transformation ratio* denoted as K, while $\left(\frac{N_1}{N_2}\right)$ known as turn ratio.

Thus, $E_2 = KE_1$ where $K = \frac{N_2}{N_1}$

1- If $N_2 > N_1$ ($i.e.$ $K > 1$), we get $E_2 > E_1$ then the transformer is called *"step-up transformer"*.

2- If $N_2 < N_1$ ($i.e.$ $K < 1$), we get $E_2 < E_1$ then the transformer is called *"step-down transformer"*.

3- If $N_2 = N_1$ ($i.e.$ $K = 1$), we get $E_2 = E_1$ then the transformer is called *"isolation transformer"* or 1: 1 *transformer*.

(*ii*) Current Ratio:

For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1, is same as the product of secondary voltage V_2 and the secondary current I_2.

So V_1I_1 = input VA & V_2I_2 = output VA

For an ideal transformer, $V_1 I_1 = V_2 I_2$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

Volt-Ampere Rating:

Transformer rating is specified as the product of voltage and current and called VA rating.

The full load primary and secondary currents which indicate the safe maximum values of currents which transformer windings can carry can be given as:

$$I_1 \text{ full load } = \frac{\text{KVA rating}\times 1{,}000}{V_1} \quad \text{(1000 to convert KVA to VA)}$$

$$I_2 \text{ full load } = \frac{\text{KVA rating}\times 1{,}000}{V_2} \quad \text{(1000 to convert KVA to VA)}$$

Example 1: The maximum flux density in the core of a 250/3,000V, 50Hz single-phase transformer is $1.2\ \text{Wb/m}^2$. If the e.m.f. per turn is 8V, determine:

(i) primary and secondary turns (ii) area of the core.

Solution:

(i) $E_1 = N_1 \times \text{e.m.f. induced/turn}$

$$N_1 = \frac{250}{8} = 31.25 \quad ; \quad N_2 = \frac{3{,}000}{8} = 375$$

(ii) We may use $E_2 = 4.44 f N_2 B_m A$

$\therefore \quad 3{,}000 = 4.44 \times 50 \times 375 \times 1.2 \times A \quad \Rightarrow\Rightarrow A = 0.03 m^2$

Example 2: A single-phase transformer has 400 primary and 1,000 secondary turns. The net cross-sectional area of the core is 60cm^2. If the primary winding be connected to a 50Hz supply at 520V, calculate (i) the peak value of flux density in the core (ii) the voltage induced in the secondary winding.

Solution:

$$K = \frac{N_2}{N_1} = \frac{1{,}000}{400} = 2.5$$

(i) $\frac{E_2}{E_1} = K \quad \Rightarrow\Rightarrow \therefore E_2 = KE_1 = 2.5 \times 520 = 1{,}300\text{V}$

(ii) $E_1 = 4.44 f N_1 B_m A$

$$520 = 4.44 \times 50 \times 400 \times B_m \times (60 \times 10^{-4})$$

$\therefore \quad B_m \cong 0.976\ \text{Wb/m}^2$

Example 3: A single phase transformer has 500 turns in the primary and 1,200 turns in the secondary. The cross-sectional area of the core is 80cm^2. If the primary winding is connected to a 50Hz supply at 500V, calculate (i) Peak flux-density, and (ii) Voltage induced in the secondary.

Solution:

From the e.m.f. equation for transformer,

$$500 = 4.44 \times 50 \times \Phi_m \times 500$$

$$\Phi_m = \frac{1}{222}\text{Wb}$$

(*i*) Peak flux density, $B_m = \frac{\Phi_m}{(80\times10^{-4})} = 0.563\,\text{Wb/m}^2$

(*ii*) Voltage induced in secondary is obtained from transformation ratio or turns ratio

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$V_2 = 500 \times \frac{1{,}200}{500} = 1{,}200\text{V}$$

Example 4: A 25kVA, single-phase transformer has 250 turns on the primary and 40 turns on the secondary winding. The primary is connected to 1,500V, 50Hz mains. Calculate (*i*) Primary and Secondary currents on full-load, (*ii*) Secondary e.m.f., (*iii*) maximum flux in the core.

Solution:

(*i*) If V_2 = Secondary voltage rating, = secondary e.m.f.,

$$\frac{V_2}{1{,}500} = \frac{40}{250}, \text{ giving } V_2 = 240\text{V}$$

(*ii*) Primary current $= \frac{25{,}000}{1{,}500} \cong 16.67\text{A}$

Secondary current $= \frac{25{,}000}{240} \cong 104.2\text{A}$

(*iii*) If Φ_m is the maximum core-flux in Wb,

$$1{,}500 = 4.44 \times 50 \times \Phi_m \times 250$$

$$\Rightarrow\Rightarrow \quad \Phi_m = 0.027\text{Wb} \quad \text{or} \quad 27\text{mWb}$$

Ideal transformer:

(*i*) No voltage drop in the windings resistance ($E_1 = V_1$ and $E_2 = V_2$).

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

(*ii*) No leakage flux *i.e.*, the same flux links both the windings.

(*iii*) There are no losses.

Therefore, volt-amperes input to the primary are equal to the output volt-amperes (input VA = output VA)

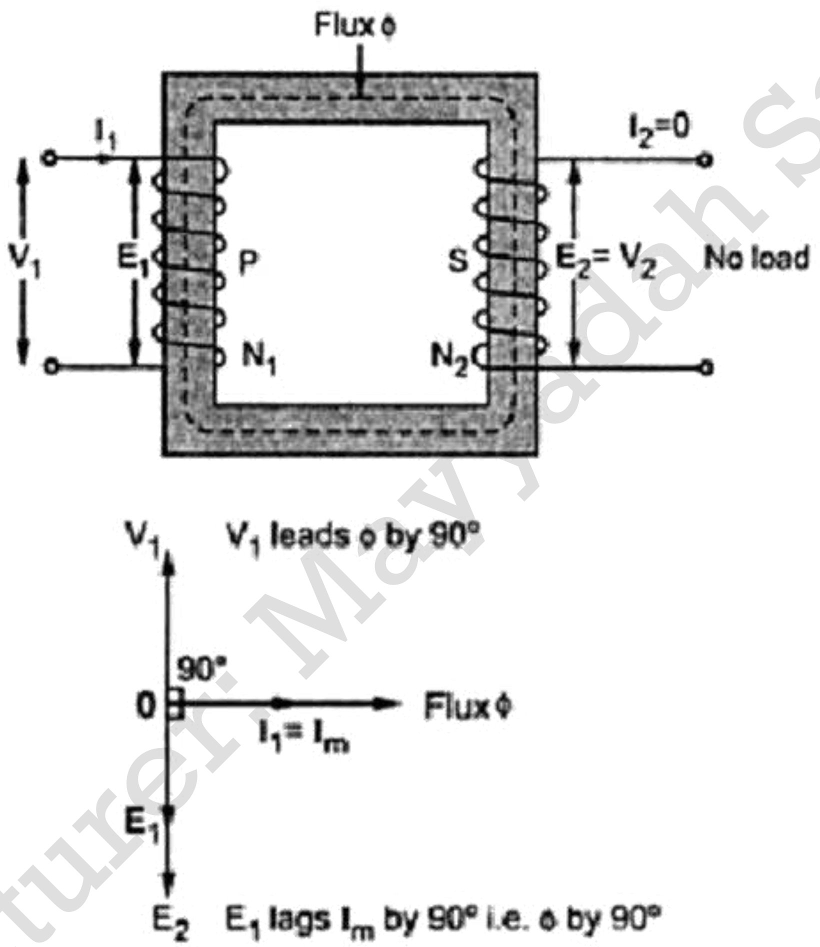
$$V_1 I_1 = V_2 I_2 \quad \text{or} \quad \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

Ideal Transformer on No Load:

Consider an ideal transformer in Figure (13-a). For no load $I_2 = 0$. I_1 is just necessary to produce flux in the core, which is called *magnetizing* current denoted as I_m. I_m is very small and lags V_1 by 90° as the winding is purely inductive.

According to Lenz's law, the induced e.m.f. opposes the cause producing it which is supply voltage V_1. Hence E_1 and E_2 are in antiphase with V_1 but equal in magnitude and E_1 and E_2 are in phase.



(a) Ideal transformer (b) The phase diagram

Figure (13)

Example 5: The primary and secondary of a 25kVA transformer has 500 and 40 turns, respectively. If the primary is connected to 3,000V, 50Hz mains, calculate (*i*) primary and secondary currents at full load ; (*ii*) The secondary e.m.f and ; (*iii*) The maximum flux in the core, assume ideal transformer.

Solution:

(*i*) At full load, $I_1 = \frac{25\times10^3}{3,000} = 8.33\text{A}$

Now, $\frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$

Secondary currents, $I_2 = \frac{N_1}{N_2} \times I_1 = \frac{500}{40} \times 8.33 = 104.125\text{A}$

(*ii*) Secondary e.m.f. $E_2 = \frac{N_2}{N_1} \times E_1 = \frac{40}{500} \times 3{,}000 = 240\text{V}$

(*iii*) Using relation, $E_1 = 4.44 N_1 f \Phi_m$

$3{,}000 = 4.44 \times 500 \times 50 \times \Phi_m$

$\Rightarrow\Rightarrow \quad \Phi_m = \frac{3{,}000}{4.44 \times 500 \times 50} \cong 27\text{mWb}$

Example 6: The Secondary of a 500kVA, 4,400/500 V, 50Hz, single-phase transformer has 500 turns. Determine (*i*) e.m.f per turn, (*ii*) primary turns, (*iii*) secondary full load current (*iv*) maximum flux, assume ideal transformer.

Solution:

Here, Rating $= 500\text{kVA}$; $E_1 = V_1 = 4{,}400\text{V}$; $E_2 = V_2 = 500\text{V}$; $f = 50\text{Hz}$; $N_2 = 500$

(*i*) e.m.f per turn $= \frac{E_2}{N_2} = \frac{500}{500} = 1.0 \text{ V/turn}$

(*ii*) e.m.f per turn $= \frac{E_2}{N_2} = \frac{E_1}{N_1} = \frac{4{,}400}{N_1} = 1.0$

$\therefore$ Primary turns, $N_1 = \frac{4{,}400}{1.0} = 4{,}400$

(*iii*) Secondary full load current, $I_2 = \frac{kVA \times 1{,}000}{V_2} = \frac{500 \times 1{,}000}{500} = 1{,}000\text{A}$

(*iv*) Maximum flux, $\Phi_m = \frac{E_2}{4.44 \times N_2 \times f} = \frac{500}{4.44 \times 500 \times 50} \cong 4.5\text{mWb}$

Example 7: A 100kVA, 3,300/200 V, 50Hz single phase transformer has 40 turns on the secondary, calculate: (*i*) the values of primary and secondary currents. (*ii*) the number of primary turns. (*iii*) the maximum value of the flux. If the transformer is to be used on a 25Hz system. (*iv*) the primary voltage, assuming that the flux is increased by 10% (*v*) the kVA rating of the transformer assuming the current density in the windings to be unaltered.

Solution:

(*i*) Full load primary current, $I_1 = \frac{100 \times 1{,}000}{3{,}300} = 30.3\text{A}$

Full load secondary current, $I_2 = \frac{100 \times 1{,}000}{200} = 500\text{A}$

(*ii*) No. of primary turns, $N_1 = N_2 \times \frac{E_1}{E_2} = 40 \times \frac{3{,}300}{200} = 660$

(*iii*) We know $E_2 = 4.44 f \Phi_{max} N_2$ V

$200 = 4.44 \times 50 \times \Phi_{max} \times 40$

$\therefore \quad \Phi_{max} = \frac{200}{4.44 \times 50 \times 40} \cong 0.0225\text{Wb}$ or 22.5mWb

(iv) As the flux is increased by 10% at 25Hz

$\therefore$ Flux at 25Hz, $\Phi_m = 0.0225 \times 1.1 = 0.02475\text{Wb}$

$\therefore$ primary voltage $= 4.44 \times N_1 \times f' \times \Phi_m \quad \text{V}$

$\therefore$ primary voltage $= 4.44 \times 660 \times 25 \times 0.02475 \cong 1{,}813\text{V}$

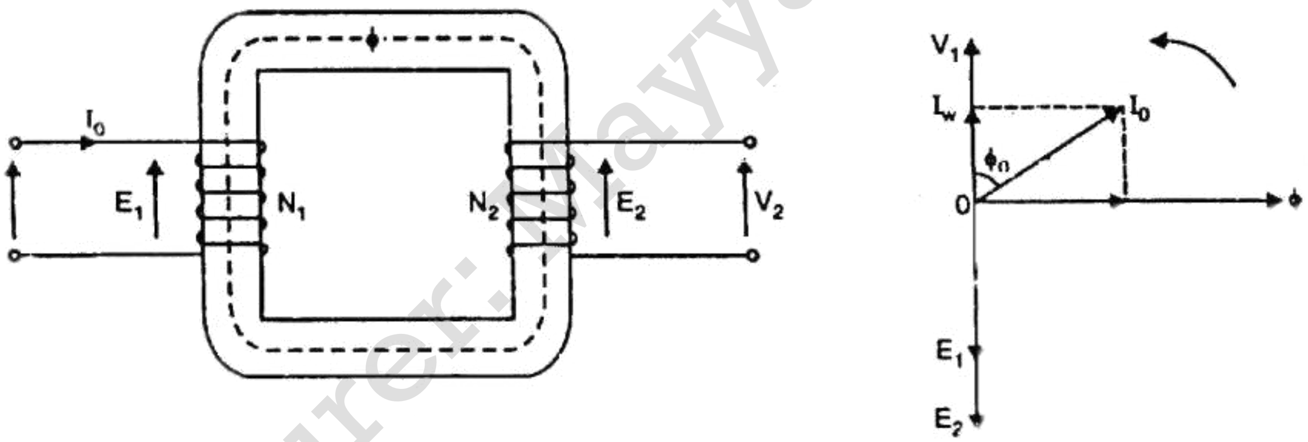
(v) For the same current density, the full load primary and secondary currents remain unaltered.

$\therefore$ kVA rating of the transformer $= \frac{30.3 \times 1{,}813}{1{,}000} \cong 55\text{kVA}$

Practical Transformer on No Load:

Consider a practical transformer on no load $i.e.$, secondary on open-circuit as shown in Figure (14-a). The primary will draw a small current I_0 to supply (i) the iron losses and (ii) a very small amount of copper loss in the primary. Hence the primary no load current I_0 is not 90° behind the applied voltage V_1 but lags it by an angle $\Phi_0 < 90°$ as shown in the phasor diagram in Figure (14-b).

No load input power, $W_0 = V_1 I_0 \cos \Phi_0$



(a) (b)

Figure (14)

As seen from the phasor diagram in Figure (14-b), the no-load primary current I_0 can be resolved into two rectangular components viz.

(a) The component I_w in phase with the applied voltage V_1. This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$I_w = I_0 \cos \Phi_0$

(b) The component I_m lagging behind V_1 by 90° and is known as magnetizing component. It is this component which produces the mutual flux Φ in the core.

Working component, $I_w = I_0 \cos \Phi_0$
Magnetising component, $I_{mag} = I_0 \sin \Phi_0$

No-load current, $I_0 = \sqrt{I_w^2 + I_{mag}^2}$

Primary p.f. at no-load, $\cos \Phi_0 = \frac{I_w}{I_0}$

No-load power input, $P_0 = V_1 I_0 \cos \Phi_0$

Exciting resistance, $R_0 = \frac{V_1}{I_w}$

Exciting reactance, $X_0 = \frac{V_1}{I_{mag}}$

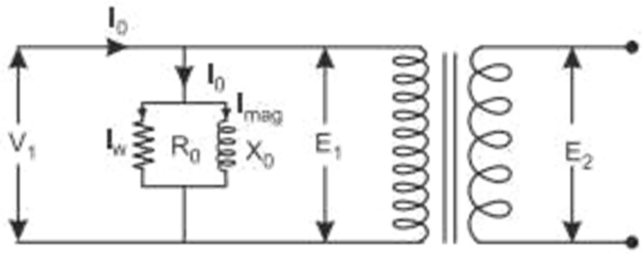


Figure (15)

It is emphasized here that no load primary copper loss ($i.e.$ I_0^2 $R1$) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer $i.e.$,

No load input power, W_0 = Iron loss

Note: At no load, there is no current in the secondary so that $V_2 = E_2$. On the primary side, the drops in R_1 and X_1, due to I_0 are also very small because of the smallness of I_0. Hence, we can say that at no load, $V_1 = E_1$.

Example 8: A 2,200/200 V transformer draws a no-load primary current of 0.6A and absorbs 400W. Find the magnetising and iron loss currents.
Solution:

Iron-loss current $= \frac{\text{no-load input in watts}}{\text{primary voltage}} = \frac{400}{2,200} \cong 0.182\text{A}$

Now, $I_0^2 = I_w^2 + I_\mu^2$

Magnetising component, $I_\mu = \sqrt{(0.6^2 - 0.182^2)} \cong 0.572\text{A}$

Example 9: A 2,200/250 V transformer takes 0.5A at a p.f. of 0.3 on open circuit. Find magnetising and working components of no-load primary current.
Solution:

$I_0 = 0.5\text{A}$; $\cos \Phi_0 = 0.3$

$\therefore \quad I_w = I_0 \cos \Phi_0 = 0.5 \times 0.3 = 0.15\text{A}$

$I_\mu = \sqrt{0.5^2 - 0.15^2} \cong 0.477\text{A}$

Example 10: A 230V, 50Hz transformer has 200 primary turns. It draws 5A at 0.25 p.f. lagging at no-load. Determine: (i) Maximum value of flux in the core ;

(*ii*) Core loss ; (*iii*) Magnetising current ; (*iv*) Exciting resistance and reactance of the transformer.

Solution:

(*i*) Using the relation, $E_1 = 4.44 N_1 f \Phi_m$

$230 = 4.44 \times 200 \times 50 \times \Phi_m$

∴ Maximum value of flux $\Phi_m = 5.18\text{mWb}$

(*ii*) Core loss, $P_0 = V_1 I_0 \cos\Phi_0 = 230 \times 5 \times 0.25 = 287.5\text{W}$

(*iii*) No-load p.f., $\cos\Phi_0 = 0.25$; $\cos^{-1} 0.25 \cong 75.52°$

$\Rightarrow\Rightarrow \sin\Phi_0 = \sin 75.52° = 0.9682$

Magnetising current component, $I_m = I_0 \sin\Phi_0 = 5 \times 0.9682 \cong 4.84\text{A}$

(*iv*) Exciting resistance, $R_0 = \frac{V_1}{I_w} = \frac{230}{I_0 \cos\Phi_0} = \frac{230}{5\times0.25} = 184\Omega$

Exciting reactance, $X_0 = \frac{V_1}{I_{mag}} = \frac{230}{4.84} = 47.52\Omega$

Example 11: A $230/110$ V single-phase transformer has a core loss of 100W. If the input under no-load condition is 400VA, find core loss current, magnetising current and no-load power factor angle.

Solution:

Here, $V_1 = 230\text{V}$; $V_2 = 110\text{V}$; $P_\text{i} = 100\text{W}$; Input at no-load = 400VA

$V_1 I_0 = 400\text{VA}$

No load current, $I_0 = \frac{400}{230} = 1.739\text{A}$

Core loss current, $I_w = \frac{P_\text{i}}{V_1} = \frac{100}{230} \cong 0.4348\text{A}$

Magnetising current, $I_{mag} = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.739)^2 - (0.4348)^2} \cong 1.684\text{A}$

No-load power factor, $\cos\Phi_0 = \frac{I_w}{I_0} = \frac{0.4348}{1.739} = 0.25$ lag

No-load power factor angle, $\Phi_0 = \cos^{-1} 0.25 \cong 75.52°$

Losses in a Transformer:

(*i*) Core or Iron Loss:

Hysteresis loss, $W_h = \eta B_{max}^{1.6} f V$ watt

Eddy current loss, $W_e = P B_{max}^2 f^2 t^2$ watt

(*ii*) Copper loss:

Total Cu loss $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$ watt

Efficiency of a Transformer:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output+losses}} = \frac{\text{Output}}{\text{Output+Cu loss+Cu loss}}$$

$$\text{Or } \eta = \frac{\text{Input}-\text{Losses}}{\text{Input}} = 1 - \frac{\text{losses}}{\text{Input}}$$

Condition for Maximum Efficiency:

Cu loss $= I_1^2 R_{01}$ or $I_2^2 R_{02} = W_{cu}$

Iron loss $=$ Hysteresis loss $+$ Eddy current loss $= W_h + W_e = W_i$

Considering primary side,

Primary input $= V_1 I_1 \cos \Phi_1$

$$\eta = \frac{V_1 I_1 \cos \Phi_1 - \text{losses}}{V_1 I_1 \cos \Phi_1} = \frac{V_1 I_1 \cos \Phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos \Phi_1} = 1 - \frac{I_1 R_{01}}{V_1 \cos \Phi_1} - \frac{W_i}{V_1 I_1 \cos \Phi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos \Phi_1} + \frac{W_i}{V_1 I_1^2 \cos \Phi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence, the above equation becomes

$$\frac{R_{01}}{V_1 \cos \Phi_1} = \frac{W_i}{V_1 I_1^2 \cos \Phi_1} \quad \text{or} \quad W_i = I_1^2 R_{01} \quad \text{or} \quad W_i = I_2^2 R_{02}$$

Or Cu loss $=$ Iron loss

The output current corresponding to maximum efficiency is $I_2 = \sqrt{(W_i / R_{02})}$

Load kVA corresponding to maximum efficiency, $=$ Full load $\times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L.Cu loss}}\right)}$

The efficiency at any load is given by $\eta = \frac{x \times \text{full}-\text{load kVA} \times \text{p.f.}}{(x \times \text{full}-\text{load kVA} \times \text{p.f.}) + W_{cu} x^2 + W_i} \times 100$

Where:

$x =$ ratio of actual to full-load in kVA

$W_i =$ iron loss in kW

$W_{cu} =$ Cu loss in kW

Why Transformer Rating in kVA?

As seen, Cu loss of a transformer depends on current and iron loss on voltage. Hence, total transformer loss depends on volt-ampere (VA) and not on phase angle between voltage and current $i.e.$ it is independent of load power factor. That is why rating of transformers is in kVA and not in kW.

Example 12: A 600kVA, 1-phase transformer has an efficiency of 92% both at full-load and half-load at unity power factor. Determine its efficiency at 60% of full-load at 0.8 power factor lag.

Solution:

$$\eta = \frac{x \times \text{full-load kVA} \times \text{p.f.}}{(x \times \text{full-load kVA} \times \text{p.f.}) + W_{cu} + W_i} \times 100 = \frac{x \times \text{kVA} \times \cos\Phi}{(x \times \text{kVA}) \times \cos\Phi + W_i + x^2 W_{cu}} \times 100$$

Where:

x = represents percentage of full-load in kVA

W_i = iron loss in kW ; W_{cu} = Cu loss in kW

At F.L. u.p.f.: Here $x = 1$

$$\therefore \quad 92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_1 + W_{cu}} \times 100, \; W_i + W_{cu} \cong 52.174\text{kW} \qquad \ldots(i)$$

At half F.L. UPF: Here $x = \frac{1}{2}$

$$92 = \frac{(1/2) \times 600 \times 1}{(1/2) \times 600 \times 1 + W_i + (1/2)^2 W_{cu}} \times 100 \;;\; W_i + 0.25W_{cu} \cong 26.087\text{kW}$$

$\ldots(ii)$

From (i) and (ii), we get, $W_i = 17.39\text{kW}$; $W_{cu} = 34.78\text{kW}$

60% F.L. 0.8 p.f. (lag) Here, $x = 0.6$

$$\eta = \frac{0.6 \times 600 \times 0.8}{(0.6 \times 600 \times 0.8) + 17.39 + (0.6)^2 \times 34.78} \times 100 = 90.59\%$$

Example 13: A 11,000/230 V, 150kVA, 1-phase, 50Hz transformer has core loss of 1.4kW and F.L. Cu loss of 1.6kW. Determine:

(i) The kVA load for max. efficiency and value of max. efficiency at unity p.f.

(ii) The efficiency at half F.L. 0.8 p.f. leading

Solution:

(i) Load kVA corresponding to maximum efficiency is

$$= \text{F.L kVA} \times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L.Cu loss}}\right)} = 150 \times \sqrt{\frac{1.4}{1.6}} \cong 140\text{kVA}$$

Since Cu loss equals iron loss at maximum efficiency,

Total loss $= 1.4 + 1.4 = 2.8\text{kW}$, Output $= 140 \times 1 = 140\text{kW}$

$$\eta_{max} = \frac{140}{142.8} \cong 0.9804 \quad \text{or} \quad 98.04\%$$

(ii) Cu loss at half full-load $= 1.6 \times \left(\frac{1}{2}\right)^2 = 0.4\text{kW}$,

Total loss $= 1.4 + 0.4 = 1.8\text{kW}$

Half F.L. output at 0.8 p.f. $= \left(\frac{150}{2}\right) \times 0.8 = 60\text{kW}$

$$\therefore \quad \text{Efficiency} = \frac{60}{60 + 1.8} = 0.97 \quad \text{or} \quad 97.08\%$$