UNIVERSITY OF DIYALA

COLLEGE OF ENGINEERING

DEPARTMENT OF THE ELECTRICAL POWER AND MACHINES ENGINEERING

Second Stage

ELECTRICAL MACHINES

Lecture: 7

Lecturer

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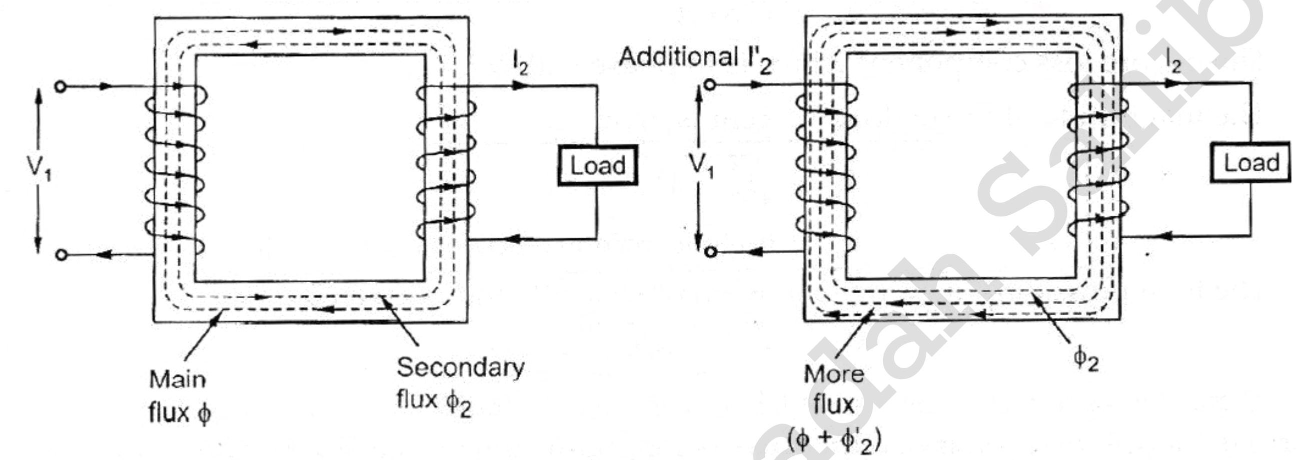
Transformer on Load:

When the transformer is loaded, the current I_2 flows through the secondary winding. The magnitude and phase of I_2 is determined by the load.
There exists a secondary m.m.f. $N_2 I_2$ due to which secondary current sets up its own flux Φ_2. This flux opposes the main flux Φ produced in the core due to magnetising component of no-load current. Hence the m.m.f. $N_2 I_2$ is called demagnetizing ampere-turns.



(a) Φ_2 opposes Φ (b) Primary draws more current

Figure (1)

The flux Φ_2 momentarily reduces the main flux Φ, due to which the primary induced e.m.f. E_1 also reduces. Hence the vector difference V_1 - E_1 increases due to which primary draws more current from the supply. This additional current drawn by primary is due to the load hence called load component of primary current denoted as I_2'.
This current I_2' is antiphase with I_2. The current I_2' sets up its own flux Φ_2' which opposes the flux Φ_2 and helps the main flux Φ. This flux Φ_2' neutralises the flux Φ_2 produced by I_2. The m.m.f. ampere turns $N_1 I_2'$ balanced the ampere turns $N_2 I_2$. Hence the net flux in the core is again maintained at constant level.
As the ampere turns are balanced we can write,

$$N_2 I_2 = N_1 I_2'$$
$$I_2' = \frac{N_2}{N_1} I_2$$
$$I_2' = K I_2$$

When transformer is loaded, the primary current I_1 has two components:
1. The no load current I_0 which lags V_1 by angle Φ_0. It has two components I_m and I_c.
2. The load components I_2' which antiphase with I_2. And phase of I_2 decided by the load.

Hence primary current I_1 is vector sum of I_0 and I_2'.

$\therefore \quad I_1 = I_0 + I_2'$

Assume inductive load, I_2 lags E_2 by Φ_2, the phasor diagram is shown in the Figure (2-a).

Assume purely resistive load, I_2 in phase with E_2, the phasor diagram is shown in the Figure (2-b).

Assume capacitive load, I_2 leads E_2 by Φ_2, the phasor diagram is shown in the Figure (2-c).

Note: that I_2' is always in antiphase with I_2.

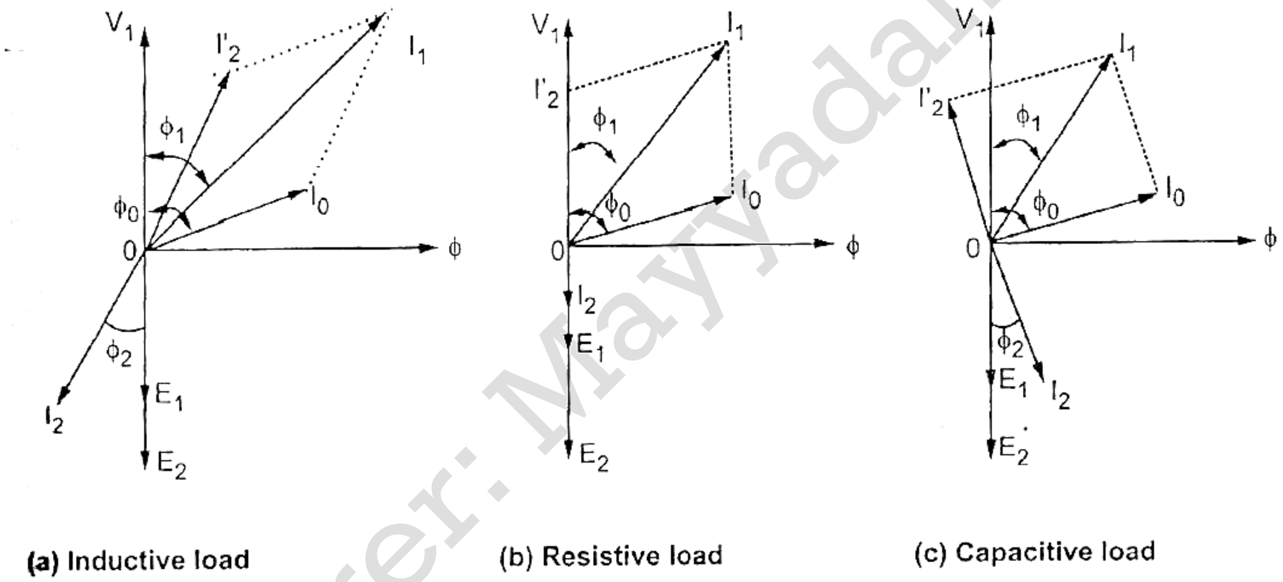


Figure (2)

But if we neglect I_0 as compared to I_2'

$N_1 I_2' = N_1 I_1 = N_2 I_2$

$\therefore \quad \frac{I_2'}{I_2} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$

Example 1: A single-phase transformer with a ratio of $440/110$ V takes a no-load current of 5A at 0.2 power factor lagging. If the secondary supplies a current of 120A at a p.f. of 0.8 lagging, estimate the current taken by the primary.

Solution:

$\cos\Phi_2 = 0.8, \ \Phi_2 = \cos^{-1} 0.8 \cong 36.87°$

$\cos\Phi_0 = 0.2, \ \Phi_0 = \cos^{-1} 0.2 = 78.46°$

Now, $K = \frac{V_2}{V_1} = \frac{110}{440} = 1/4$

$\therefore \quad I_2' = KI_2 = 120 \times 1/4 = 30\text{A}$

$I_0 = 5\text{A}$

Angle between I_0 and I_2'

$= 78.46° - 36.87° = 41.59°$

Using parallelogram law of vectors Figure (3) we get

$I_1 = \sqrt{(5^2 + [2 \times 5 \times 30 \times \cos 41.59°] + 30^2)}$

$= 33.9\text{A}$

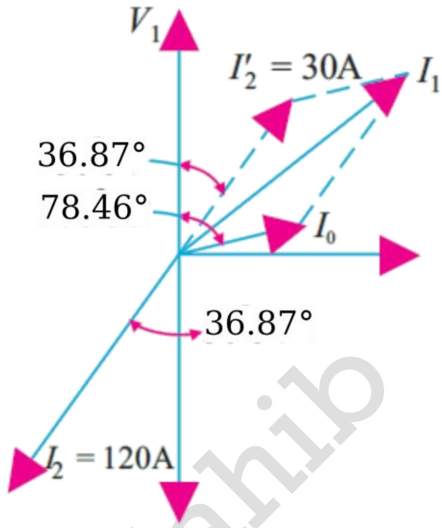


Figure (3)

The resultant current could also have been found by Resolving I_2' and I_0 into their X and Y-components.

Note: vectors can be solved by any one of these methods,

1. Parallelogram method (as in Example 1).
2. Vector components analysis.
3. Complex number methods.

Example 2: A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80A at 0.8 power factor lagging, the primary current is 25A at 0.707 power factor lagging. Determine graphically or otherwise the no-load current of the transformer and its phase with respect to the voltage.

Solution:

Here, $K = 200/800 = 1/4$; $I_2' = 80 \times 1/4 = 20\text{A}$; $\Phi_2 = \cos^{-1} 0.8 \cong 36.9°$;

$\Phi_1 = \cos^{-1} 0.707 \cong 45°$

As seen from Figure (4), I_1 is the vector sum of I_0 and I_2'. Let I_0 lag behind V_1 by an angle Φ_0.

$I_0 \cos \Phi_0 + 20 \cos 36.9° = 25 \cos 45°$

$\therefore \quad I_0 \cos \Phi_2 = 25 \times 0.707 - 20 \times 0.8$

$I_0 \cos \Phi_2 = 1.675\text{A}$

$I_0 \sin \Phi_0 + 20 \sin 36.9° = 25 \sin 45°$

$\therefore \quad I_0 \sin \Phi_0 = 25 \times 0.707 - 20 \times 0.6$

$I_0 \sin \Phi_0 = 5.675\text{A}$

$\therefore \quad \tan \Phi_0 = \frac{5.675}{1.675} = 3.388$

$\therefore \quad \Phi_0 \cong 73.6°$

Now, $I_0 \sin \Phi_0 = 5.675$

$\therefore \quad I_0 = \frac{5.675}{\sin 73.6°} \cong 5.92\text{A}$

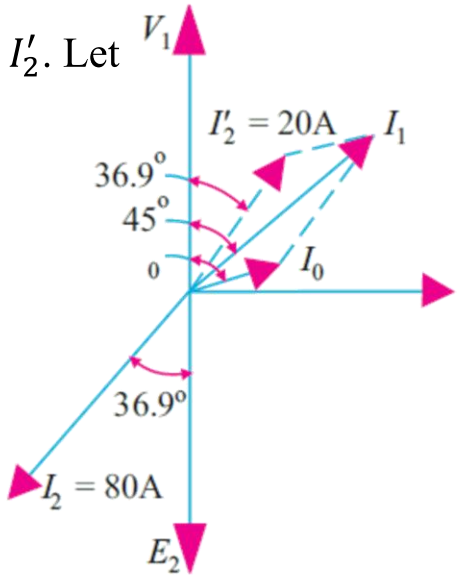


Figure (4)

Example 3: A single phase transformer takes 10A on no load at p.f. of 0.2 lagging. The turns ratio is 4: 1 (step down). If the load on the secondary is 200A at a p.f. of 0.85 lagging. Find the primary current and power factor.
Neglect the voltage-drop in the winding.

Solution:
Secondary load of 200A, 0.85 lag is reflected as 50A, 0.85 lag in terms of the primary equivalent current.

$I_0 = 10\angle - \Phi_0$, where $\Phi_0 = \cos^{-1} 0.20 = 78.5°$ lagging

$I_0 \cong 2 - j9.8A$

$I_2' = 50\angle - \Phi_L$, where $\Phi_L = \cos^{-1} 0.85 = 31.8°$ lagging

$I_2' \cong 42.5 - j26.34$

Hence primary current I_1

$I_1 = I_0 + I_2'$

$I_1 = 2 - j9.8 + 42.5 - j26.34$

$I_1 = 44.5 - j36.14$

$|I_1| \cong 57.33A$; $\cos\Phi = 0.776$ lagging

$\Phi = \cos^{-1}\frac{44.5}{57.33} \cong 39.09°$ lagging

The phasor diagram is shown in Figure (5).

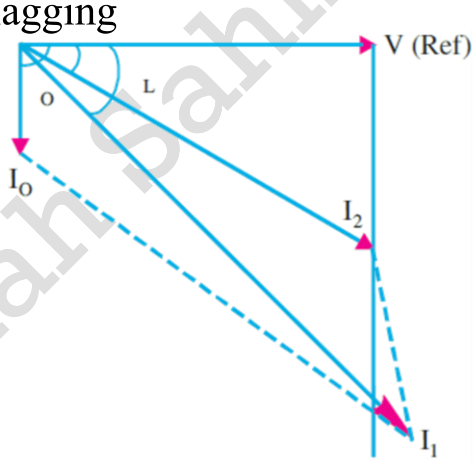


Figure (5)

Transformer with Winding Resistance but No Magnetic Leakage:

An ideal transformer was supposed to possess no resistance, but in an actual transformer, there is always present some resistance of the primary and secondary windings. Due to this resistance, there is some voltage drop in the two windings. The result is that:

(*i*) The secondary terminal voltage V_2 is vectorially less than the secondary induced e.m.f. E_2 by an amount I_2R_2 where R_2 is the resistance of the secondary winding. Hence, V_2 is equal to the vector difference of E_2 and resistive voltage drop I_2R_2.

∴ $V_2 = E_2 - I_2R_2$ ...vector difference

(*ii*) Similarly, primary induced e.m.f. E_1 is equal to the vector difference of V_1 and I_1R_1 where R_1 is the resistance of the primary winding.

$E_1 = V_1 - I_1R_1$ ...vector difference

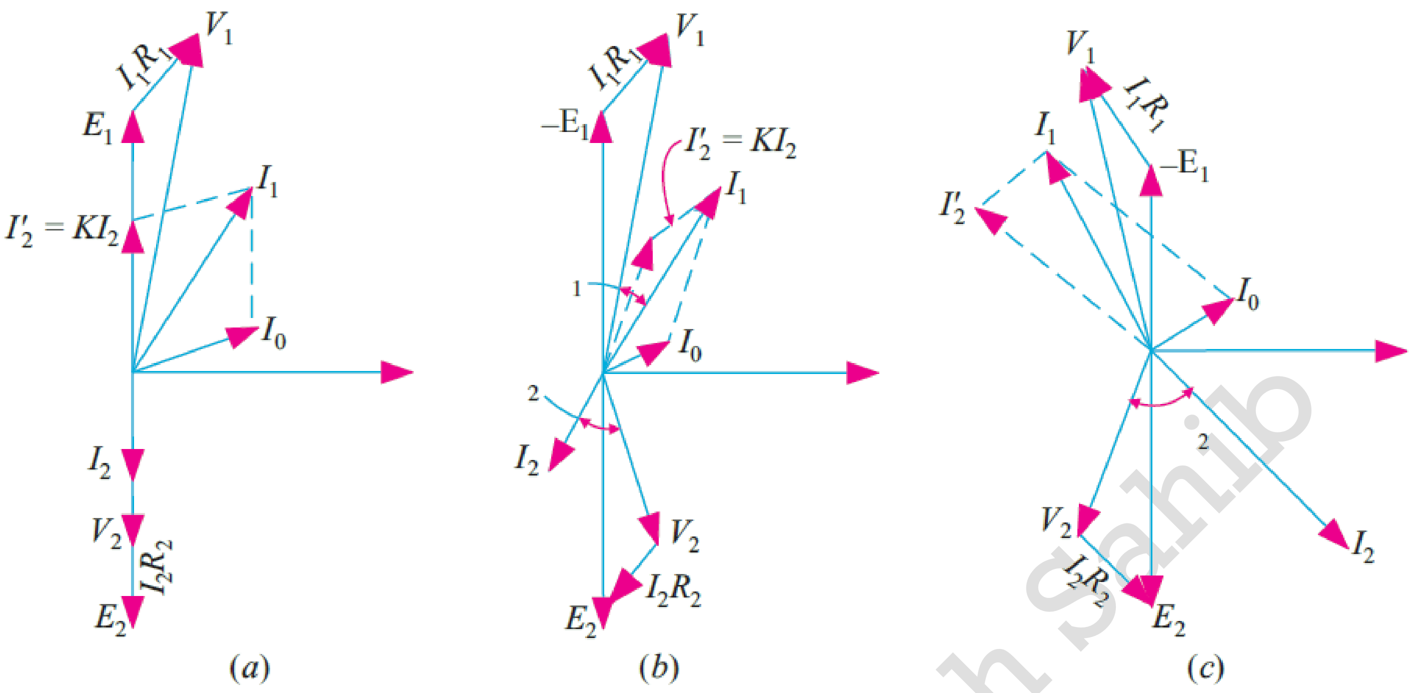


Figure (6)

The vector diagrams for non-inductive, inductive and capacitive loads are shown in Figure (6) (a), (b) and (c) respectively.

Equivalent Resistance:

In Figure (7) a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2 respectively. It would now be shown that the resistances of the two windings can be transferred to any one of the two windings. The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only.

It is to be noted that

1. a resistance of R_1 in primary is equivalent to K^2R_1 in secondary. Hence, it is called ***equivalent primary resistance as referred to secondary*** $i.e.$ R'_1 (as shown in Figure (8)).
2. a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary. Hence, it is called the ***equivalent secondary resistance as referred to primary*** $i.e.$ R'_2 (as shown in Figure (7)).
3. ***Total or effective resistance of the transformer as referred to primary*** is,

R_{01} = primary resistance + equivalent secondary resistance as referred to primary

$$R_{01} = R_1 + R'_2 = R_1 + R_2/K^2$$

4. Similarly, total ***transformer resistance as referred to secondary*** is,

R_{02} = secondary resistance + equivalent primary resistance as referred to secondary

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1$$

5. The copper loss in secondary is $(I_2)^2 R_2$. This loss is supplied by primary which takes a current of I_1. Hence if R_2' is the ***equivalent resistance in primary which would have caused the same loss*** as R_2 in secondary, then,

$$I_1^2 R_2' = I_2^2 R_2 \quad \text{or} \quad R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

Now, if *we neglect no-load current* I_0, then $I_2/I_1 = 1/K$. Hence, $R_2' = R_2/K_2$. Similarly, equivalent primary resistance as referred to secondary is $R_1' = K_2 R_1$

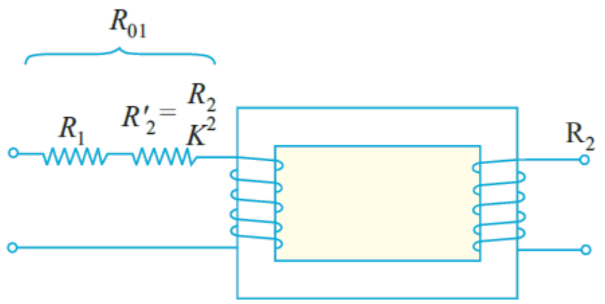


Figure (7)

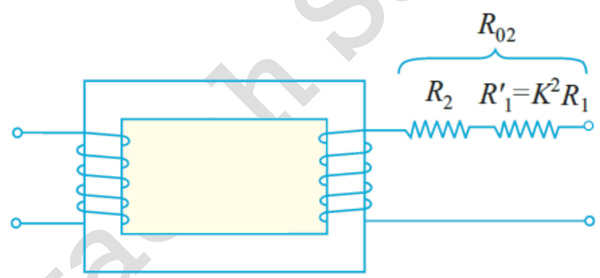


Figure (8)

6. When shifting any **voltage** from one winding to another only ***K*** is used,

$$E_1' = KE_1 \quad \& \quad E_2' = E_2/K$$

7. When shifting any **current** from one winding to another only ***K*** is used but inverse to the transformation of resistances and currents,

$$I_1' = I_1/K \quad \& \quad I_2' = KI_2$$

8. It is important to remember that,

When shifting any primary resistance to the secondary, *multiply* it by K^2 $i.e.$ (transformation ratio)^2. When shifting secondary resistance to the primary, *divide* it by K^2.

The result can be cross-checked by another approach. The high voltage winding is always low current winding and hence the resistance of high voltage side is high. The low voltage side is high current side and hence resistance of low voltage side is low. So, while transferring resistance from low voltage side to high voltage side, its value must increase while transferring resistance from high voltage side to low voltage side, its value must decrease.

High voltage side	Low current side → High resistance side
→	
Low voltage side	High current side → Low resistance side

Transformer with Resistance and Leakage Reactance:

In Figure (9) the primary and secondary windings of a transformer with reactance's taken out of the windings are shown. The primary impedance is given by

$$Z_1 = \sqrt{(R_1^2 + X_1^2)}$$

Similarly, secondary impedance is given by

$$Z_2 = \sqrt{(R_2^2 + X_2^2)}$$

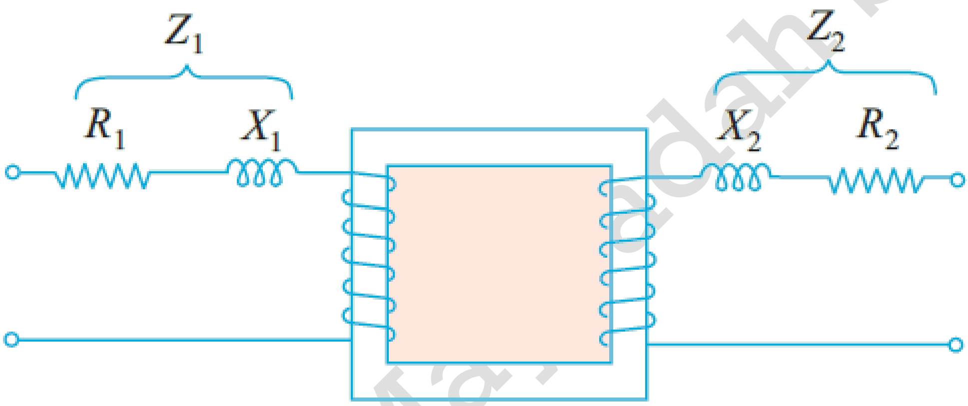


Figure (9)

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is I_1X_1 (usually 1 or 2% of V_1). Hence

$$V_1 = E_1 + I_1(R_1 + jX_1) = E_1 + I_1Z_1$$

Similarly, there are I_2R_2 and I_2X_2 drops in secondary which combine with V_2 to give E_2.

$$E_2 = V_2 + I_2(R_2 + jX_2) = V_2 + I_2Z_2$$

The vector diagram for such a transformer for different kinds of loads is shown in Figure (10). In these diagrams, vectors for resistive drops are drawn parallel to current vectors whereas reactive drops are perpendicular to the current vectors. The angle Φ_1 between V_1 and I_1 gives the power factor angle of the transformer.

It may be noted that leakage reactance's can also be transferred from one winding to the other in the same way as resistance.

$$\therefore \quad X_2' = X_2/K^2 \quad \text{and} \quad X_1' = K^2X_1$$

And $X_{01} = X_1 + X_2' = X_1 + X_2/K^2$

And $X_{02} = X_2 + X_1' = X_2 + K^2 X_1$

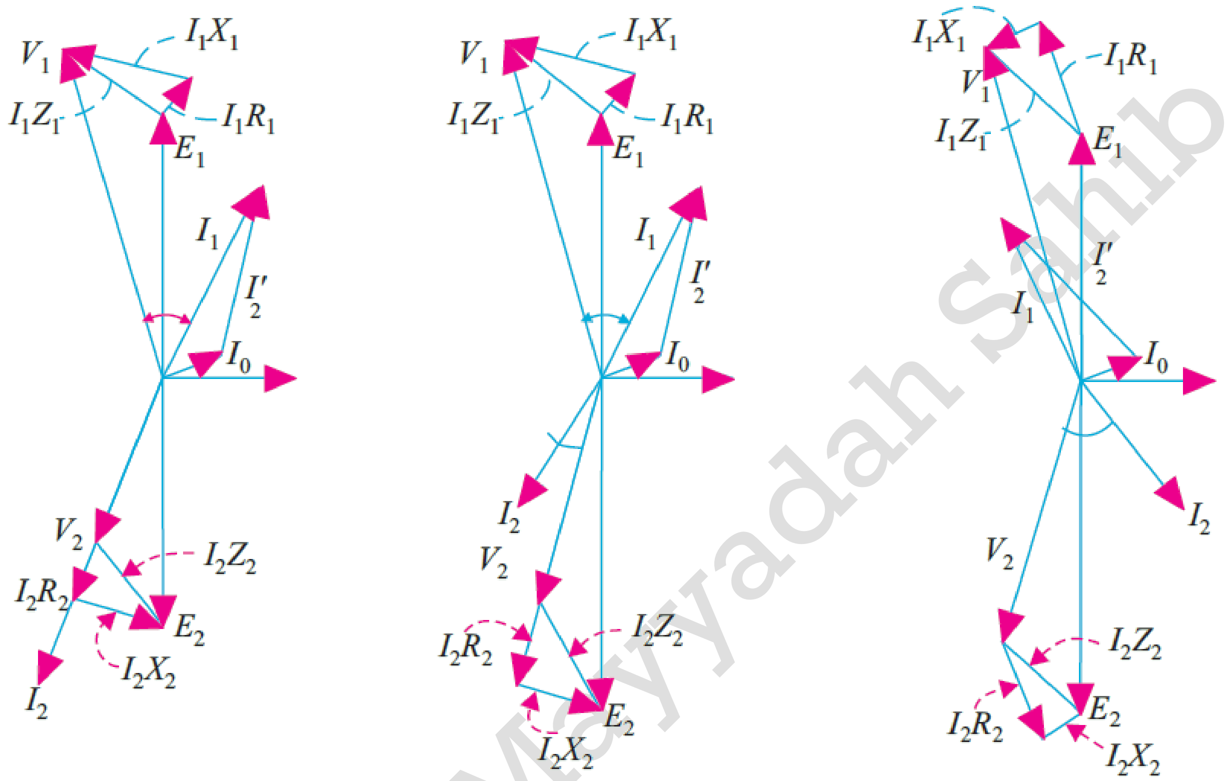
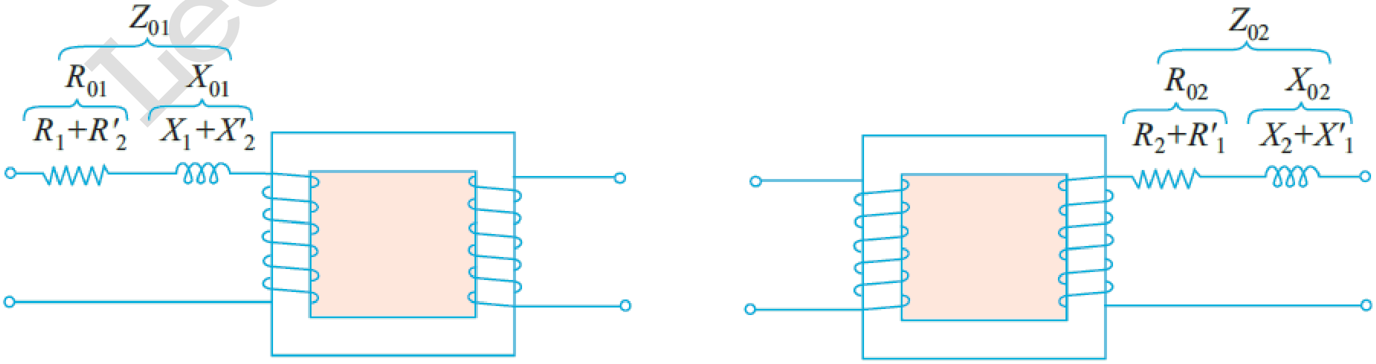


Figure (10)

It is obvious that total impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{(R_{01}^2 + X_{01}^2)} \qquad \text{Figure (11-a)}$$
$$Z_{02} = \sqrt{(R_{02}^2 + X_{02}^2)} \qquad \text{Figure (11-b)}$$



(a) (b)

Figure (11)

Example 4: A 30kVA, 2,400/120 V, 50Hz transformer has a high voltage winding resistance of 0.1Ω and a leakage reactance of 0.22Ω. The low voltage winding resistance 0.035Ω and the leakage reactance is 0.012Ω. Find the equivalent winding resistance, reactance and impedance referred to the (*i*) high voltage side and (*ii*) the low-voltage side.

Solution:

$K = 120/2{,}400 = 1/20$; $R_1 = 0.1\Omega$; $X_1 = 0.22\Omega$; $R_2 = 0.035\Omega$; $X_2 = 0.012\Omega$

(*i*) Here, high-voltage side is, obviously, the primary side. Hence, values as referred to primary side are

$$R_{01} = R_1 + R_2' = R_1 + R_2/K^2 = 0.1 + \frac{0.035}{(1/20)^2} = 14.1\Omega$$

$$X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 0.22 + \frac{0.012}{(1/20)^2} = 5.02\Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} \cong 15\Omega$$

(*ii*) $R_{02} = R_2 + R_1' = R_2 + K^2R_1 = 0.035 + (1/20)^2 \times 0.1 = 0.03525\Omega$

$$X_{02} = X_2 + X_1' = X_2 + K^2X_1 = 0.012 + (1/20)^2 \times 0.22 = 0.01255\Omega$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.03525^2 + 0.01255^2} = 0.0374\Omega$$

Or $Z_{02} = K^2Z_{01} = (1/20)^2 \times 15 = 0.0375\Omega$

Example 5: A 50kVA, 4,400/220 V transformer has $R_1 = 3.45\Omega$, $R_2 = 0.009\Omega$. The values of reactance's are $X_1 = 5.2\Omega$ and $X_2 = 0.015\Omega$. Calculate for the transformer (*i*) equivalent resistance as referred to primary (*ii*) equivalent resistance as referred to secondary (*iii*) equivalent reactance as referred to both primary and secondary (*iv*) equivalent impedance as referred to both primary and secondary (*v*) total Cu loss, first using individual resistances of the two windings and secondly, using equivalent resistances as referred to each side.

Solution:

Full-load $I_1 = \frac{50{,}000}{4{,}400} = 11.36$A (assuming 100% efficiency)

Full-load $I_2 = \frac{50{,}000}{220} \cong 227$A ; $K = \frac{220}{4{,}400} = \frac{1}{20} = 0.05$

(*i*) $R_{01} = R_1 + R_2' = R_1 + R_2/K^2 = 3.45 + \frac{0.009}{(1/20)^2} = 3.45 + 3.6 = 7.05\Omega$

(*ii*) $R_{02} = R_2 + R_1' = R_2 + K^2R_1$

$$R_{02} = 0.009 + (1/20)^2 \times 3.45 = 0.009 + 0.0086 = 0.0176\Omega$$

Also, $R_{02} = K^2R_{01} = (1/20)^2 \times 7.05 = 0.0176\Omega$

(*iii*) $X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 5.2 + \frac{0.015}{(1/20)^2} = 11.2\Omega$

$$X_{02} = X_2 + X_1' = X_2 + K^2X_1 = 0.015 + 5.2 \times (1/20)^2 = 0.028\Omega$$

Also, $X_{02} = K^2X_{01} = \frac{11.2}{400} = 0.028\Omega$

(iv) $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(7.05^2 + 11.2^2)} = 13.23\Omega$

$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{(0.0176^2 + 0.028^2)} \cong 0.0331\Omega$

Also, $Z_{02} = K^2 Z_{01} = \frac{13.23}{400} \cong 0.0331\Omega$

(v) Cu loss $= I_1^2 R_2 + I_2^2 R_2 = 11.36^2 \times 3.45 + 227^2 \times 0.009 \cong 910\text{W}$

Also, Cu loss $= I_1^2 R_{01} = 11.36^2 \times 7.05 \cong 910\text{W}$

Also, Cu loss $= I_2^2 R_{02} = 227^2 \times 0.0176 \cong 910\text{W}$

Example 6: A 100kVA, 1,100/220 V, 50Hz, single-phase transformer has a leakage impedance of $(0.1 + 0.4)\Omega$ for the H.V. winding and $(0.006 + 0.015)\Omega$ for the L.V. winding. Find the equivalent winding resistance, reactance and impedance referred to the H.V. and L.V. sides.

Solution:

$$K = \frac{V_2}{V_1} = \frac{220}{1{,}100} = 0.2 \quad \Rightarrow\Rightarrow \quad K^2 = 0.04$$

$$R_{1\,eq} = R_1 + R_2' = R_1 + R_2/K^2 = 0.1 + \frac{0.006}{0.04} = 0.1 + 0.15 = 0.25\Omega$$

$$X_{1\,eq} = X_1 + X_2' = R_1 + X_2/K^2 = 0.4 + \frac{0.015}{0.04} = 0.4 + 0.375 = 0.775\Omega$$

$$Z_{1\,eq} = \sqrt{\left(R_{1\,eq}\right)^2 + \left(X_{1\,eq}\right)^2} = \sqrt{(0.25)^2 + (0.775)^2} = 0.814\Omega$$

$$R_{2\,eq} = R_2 + R_1' = R_2 + R_1 \times K^2 = 0.006 + (0.1 \times 0.04) = 0.01\Omega$$

$$X_{2\,eq} = X_2 + X_1' = X_2 + X_1 \times K^2 = 0.015 + (0.4 \times 0.04) = 0.031\Omega$$

$$Z_{2\,eq} = \sqrt{\left(R_{2\,eq}\right)^2 + \left(X_{2\,eq}\right)^2} = \sqrt{(0.01)^2 + (0.031)^2} \cong 0.0326\Omega$$

Approximate Voltage Drop:

When the transformer is on *no-load*, then V_1 is approximately equal to E_1. Hence $E_2 = KE_1 = KV_1$. Also, $E_2 = {}_0V_2$ where ${}_0V_2$ is secondary terminal voltage on *no-load*, hence no-load secondary terminal voltage is KV_1. The secondary voltage on load is V_2. The difference between the two is $I_2 Z_{02}$ as shown in Figure (16). The approximate voltage drop of the transformer *as referred to secondary* is found thus:

$$E_2 = V_2 + I_2 R_{02} + I_2 X_{02} = V_2 + I_2(R_{02} + jX_{02}) = V_2 + I_2 Z_{02}$$

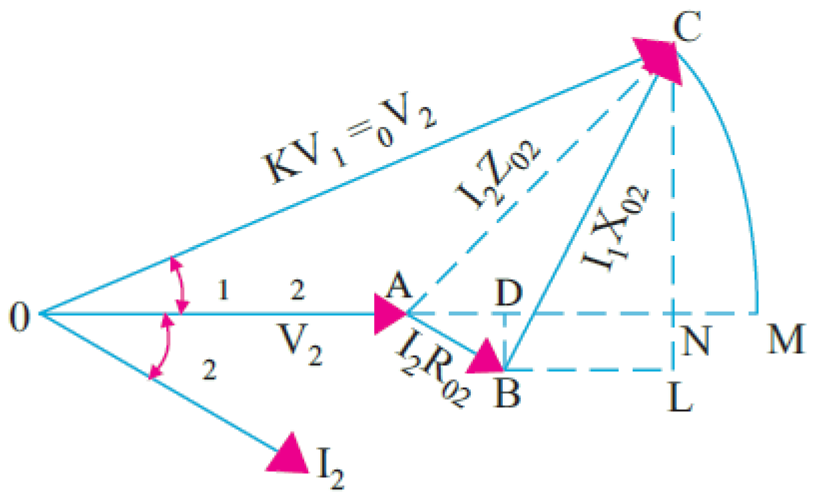


Figure (16)

With O as the centre and radius OC draw an arc cutting OA produced at M. The total voltage drop $I_2Z_{02} = AC = AM$ which is approximately equal to AN. From B draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

Approximate voltage drop

$$= AN = AD + DN$$

$$= I_2R_{02}\cos\Phi + I_2X_{02}\sin\Phi$$

Where, $\Phi_1 = \Phi_2 = \Phi$ (approx).

This is the value of approximate voltage drop for a *lagging* power factor.

The different figures for unity and leading power factors are shown in Figure (17) (a) and (b) respectively.

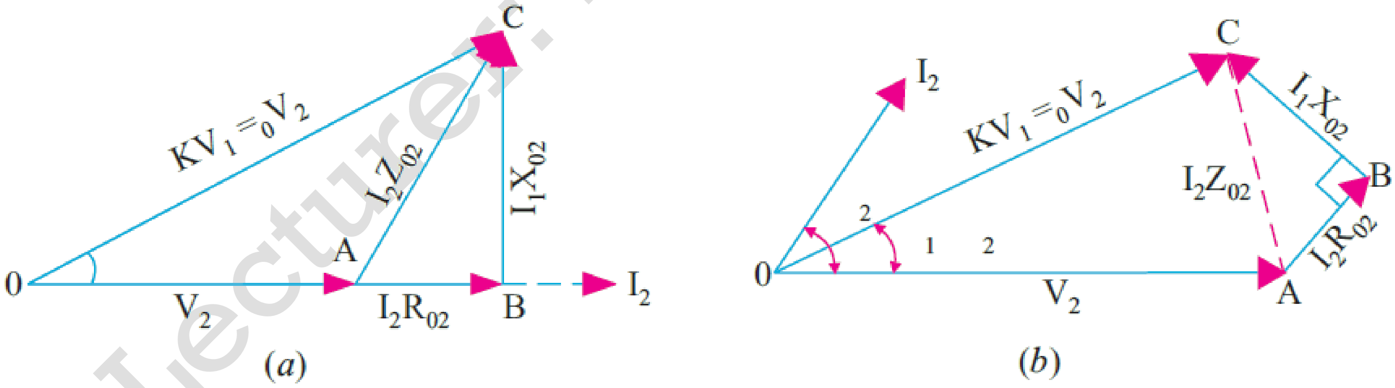


Figure (17)

The approximate voltage drop for *leading* power factor becomes

$$I_2R_{02}\cos\Phi \pm I_2X_{02}\sin\Phi$$

In general, approximate voltage drop is

$$I_2R_{02}\cos\Phi \pm I_2X_{02}\sin\Phi$$

It may be noted that approximate voltage drop as referred to primary is

$$I_1R_{01}\cos\Phi \pm I_1X_{01}\sin\Phi$$

% voltage drop in secondary is $= \frac{I_2R_{02}\cos\Phi \pm I_2X_{02}\sin\Phi}{{}_0V_2} \times 100$

$$= \frac{100 \times I_2R_{02}}{{}_0V_2}\cos\Phi \pm \frac{100 \times I_2X_{02}}{{}_0V_2}\sin\Phi$$

$$= v_r\cos\Phi \pm v_x\sin\Phi$$

Where, $v_r = \frac{100I_2R_{02}}{{}_0V_2}$ = percentage resistive drop $= \frac{100I_1R_{01}}{V_1}$

Where, $v_x = \frac{100I_2X_{02}}{{}_0V_2}$ = percentage reactive drop $= \frac{100I_1X_{01}}{V_1}$

Voltage Regulation:

The voltage regulation of a transformer is the arithmetic difference (not phasor difference) between the no-load secondary voltage (${}_0V_2$) and the secondary voltage V_2 on load expressed as percentage of no-load voltage $i.e.$

$$\%\text{ age voltage regulation} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$$

Where, ${}_0V_2$ = No-load secondary voltage $= KV_1$

Where, V_2 = Secondary voltage on load

$${}_0V_2 - V_2 = I_2R_{02}\cos\Phi_2 \pm I_2X_{02}\sin\Phi_2$$

The + ve sign is for lagging p.f. and − ve sign for leading p.f.

Transformer Tests:

The circuit constants, efficiency and voltage regulation of a transformer can be determined by two simple tests:

(*i*) Open-Circuit Test ; (*ii*) Short-Circuit Test

(*i*) Open-Circuit or No-Load Test:

This test is conducted to determine:

- The iron losses (or core losses).
- Parameters R_0 and X_0 of the transformer

In this test (in Figure (18)), the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left open-circuited. As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core. Cu losses in the primary under no-load condition are negligible as compared with iron losses.

- Iron losses, P_i = Wattmeter reading $= W_0$

- No load current = Ammeter reading = I_0
- Applied voltage = Voltmeter reading = V_1
- Input power, $W_0 = V_1 I_0 \cos\Phi_0$

No-load p.f. $\cos\Phi_0 = \frac{W_0}{V_1 I_0}$

$I_w = I_0 \cos\Phi_0$

$I_m = I_0 \sin\Phi_0$

$R_o = \frac{V_1}{I_w}$

$X_0 = \frac{V_1}{I_m}$

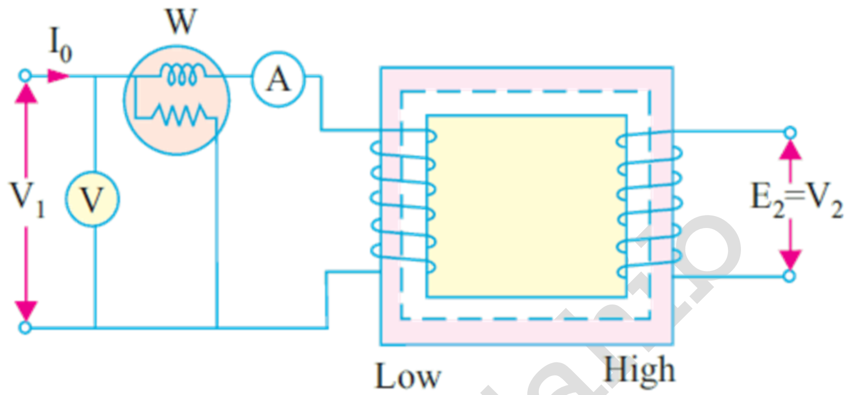


Figure (18)

(*ii*) Short-Circuit or Impedance Test:

This test is conducted to determine:

- Full-load copper losses of the transformer
- R_{01}(or R_{02}) ; X_{01}(or X_{02})

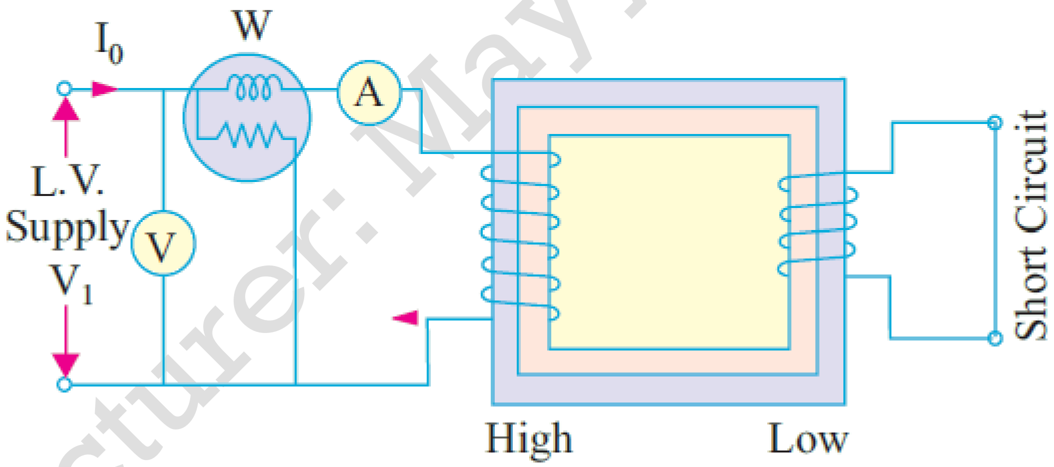


Figure (19)

In this test (in Figure (19)), the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary. The low input voltage is gradually raised till at voltage V_{SC}, full-load current I_1 flows in the primary. Then I_2 in the secondary also has full-load value since $(I_1/I_2 = N_2/N_1)$. Under such conditions, the copper loss in the windings is the same as that on full load.

For the Figure (19):

- Full load Cu loss, P_C = Wattmeter reading = W_S
- Applied voltage = Voltmeter reading = V_{SC}
- F.L. primary current = Ammeter reading = I_1

$$P_C = I_1^2 R_1 + I_1^2 R_2' = I_1^2 R_{01}$$

$$\therefore \quad R_{01} = \frac{P_C}{I_1^2}$$

Where, R_{01} is the total resistance of transformer referred to primary.

Total impedance referred to primary, $Z_{01} = \frac{V_{SC}}{I_1}$

Total leakage reactance referred to primary, $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$

Short-circuit p.f., $\cos\Phi_2 = \frac{P_C}{V_{SC} I_1}$

Example 7: Obtain the equivalent circuit of a 200/400 V, 50Hz, 1-phase transformer from the following test data:

$O.C.$ test: 200V ; 0.7A ; 70W – on L.V. side

$S.C.$ test: 15V ; 10A ; 85W – on H.V. side

Calculate the secondary voltage when delivering 5kW at 0.8 p.f. lagging, the primary voltage being 200V.

Solution:

From $O.C.$ Test:

$$V_1 I_0 \cos\Phi_0 = W_0$$

$$\therefore \quad 200 \times 0.7 \times \cos\Phi_0 = 70$$

$$\cos\Phi_0 = 0.5 \quad \text{and} \quad \sin\Phi_0 = \frac{\sqrt{3}}{2} = 0.866$$

$$I_w = I_0 \cos\Phi_0 = 0.7 \times 0.5 = 0.35\text{A}$$

$$I_\mu = I_0 \sin\Phi_0 = 0.7 \times 0.866 = 0.606\text{A}$$

$$R_0 = V_1/I_w = \frac{200}{0.35} = 571.4\Omega$$

$$X_0 = V_1/I_\mu = \frac{200}{0.606} \cong 330\Omega$$

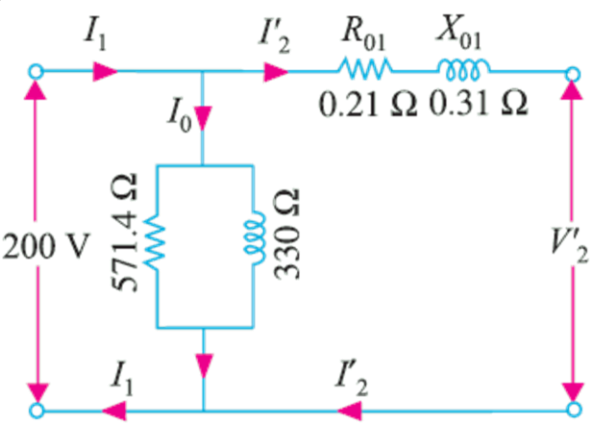


Figure (20)

As shown in Figure (20), these values refer to primary $i.e.$ low-voltage side.

From $S.C.$ Test:

It may be noted that in this test, instruments have been placed in the secondary $i.e.$ high-voltage winding whereas the low-voltage winding $i.e.$ primary has been short-circuited.

$$Z_{02} = V_{SC}/I_2 = \frac{15}{10} = 1.5\Omega \quad ; \quad K = \frac{400}{200} = 2$$

$$Z_{01} = Z_{02}/K^2 = \frac{1.5}{4} = 0.375\Omega$$

Also, $I_2^2 R_{02} = W$; $R_{02} = \frac{85}{100} = 0.85\Omega$

$$R_{01} = R_{02}/K^2 = \frac{0.85}{4} \cong 0.21\Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{0.375^2 - 0.21^2} = 0.31\Omega$$

Output kVA $= \frac{5}{0.8}$; Output current $I_2 = \frac{5{,}000}{0.8\times400} = 15.625\Omega$

This value of I_2 is approximate because V_2 (which is to be calculated as yet) has been taken equal to 400V (which, in fact, is equal to E_2 or ${}_0V_2$).

Now, $Z_{02} = 1.5\Omega$; $R_{02} = 0.85\Omega \Rightarrow\Rightarrow \therefore \quad X_{02} = \sqrt{1.5^2 - 0.85^2} \cong 1.24\Omega$

Total transformer drop as referred to secondary

$$= I_2(R_{02}\cos\Phi_2 + X_{02}\sin\Phi_2) = 15.6(0.85\times0.8 + 1.24\times0.6) = 22.2\text{V}$$

$\therefore \quad V_2 = 400 - 22.2 = 377.8\text{V}$

Example 8: A 1-phase, 10kVA, 500/250 V, 50Hz transformer has the following constants:

Reactance: primary 0.2Ω ; secondary 0.5Ω

Resistance: primary 0.4Ω ; secondary 0.1Ω

Resistance of equivalent exciting circuit referred to primary, $R_0 = 1{,}500\Omega$

Reactance of equivalent exciting circuit referred to primary, $X_0 = 750\Omega$

What would be the reading of the instruments when the transformer is connected for the open- circuit and short-circuit tests?

Solution:

$O.C.$ Test:

$$I_\mu = V_1/X_0 = \frac{500}{750} = \frac{2}{3}\text{A} \quad ; \quad I_w = V_1/R_0 = \frac{500}{1{,}500} = \frac{1}{3}\text{A}$$

$$\therefore \quad I_0 = \sqrt{\left[\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right]} = 0.745\text{A}$$

No-load primary input $= V_1 I_w = 500 \times \frac{1}{3} \cong 167\text{W}$

Instruments used in primary circuit are: voltmeter, ammeter and wattmeter, their readings being 500V ; 0.745A and 167W respectively.

$S.C.$ Test:

Suppose $S.C.$ test is performed by short-circuiting the $l.v.$ winding $i.e.$ the secondary so that all instruments are in primary.

$$R_{01} = R_1 + (R_2/K^2)$$

Here, $K = \frac{1}{2}$, $K^2 = 0.25$

$\therefore \quad R_{01} = 0.4 + (0.1/0.25) = 0.4 + 0.4 = 0.8\Omega$

Similarly, $X_{01} = X_1 + (X_2/K^2)$

$X_{01} = 0.2 + (0.5/0.25) = 0.2 + 2 = 2.2\Omega$

$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

$Z_{01} = \sqrt{(0.8^2 + 2.2^2)} \cong 2.341\Omega$

Full-load primary current

$I_1 = \frac{10,000}{500} = 20A$

$\therefore \quad V_{SC} = I_1 Z_{01} = 20 \times 2.341 = 46.82V$

Power absorbed $= I_1^2 R_{01} = 20^2 \times 2.2 = 880W$

Primary instruments will read: 46.82V ; 20A ; 880W

Example 9: Consider a 20kVA, 2,200/220 V, 50Hz transformer. The $O.C./S.C.$ test results are as follows:

$O.C.$ test: 220V ; 4.2A ; 148W ($l.v.$ side)

$S.C.$ test : 86V ; 10.5A ; 360W ($h.v.$ side)

Determine the regulation at 0.8 p.f. lagging and at full load. What is the p.f. on short-circuit?

Solution: It may be noted that $O.C.$ data is not required in this question for finding the regulation. Since during $S.C.$ test instruments have been placed on the h.v. side $i.e.$ primary side.

$\therefore \quad Z_{01} = \frac{86}{10.5} = 8.19\Omega$; $R_{01} = \frac{360}{10.5^2} \cong 3.27\Omega$; $X_{01} = \sqrt{8.19^2 - 3.26^2} = 7.51\Omega$

F.L. primary current, $I_1 = \frac{20,000}{2,200} = 9.09A$

Total voltage drop as referred to primary $= I_1(R_{01}\cos\Phi + X_{01}\sin\Phi)$

$= 9.09 \times (3.26 \times 0.8 + 7.51 \times 0.6) \cong 64.67V$

% age regn. $= \frac{64.67}{2,200} \times 100 \cong 2.94\%$; p.f. on short-circuit $= R_{01}/Z_{01} = \frac{3.26}{8.19}$

$\cong$ 0.4 lag

Losses in a Transformer:

(*i*) Core or Iron Loss:

Hysteresis loss, $W_h = \eta B_{max}^{1.6} f V$ watt

Eddy current loss, $W_e = P B_{max}^2 f^2 t^2$ watt

(*ii*) Copper loss:

Total Cu loss $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$ watt

Efficiency of a Transformer:

Efficiency $= \frac{\text{Output}}{\text{Input}}$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output+losses}} = \frac{\text{Output}}{\text{Output+Cu loss+Cu loss}}$$

Or $\eta = \frac{\text{Input}-\text{Losses}}{\text{Input}} = 1 - \frac{\text{losses}}{\text{Input}}$

Condition for Maximum Efficiency:

Cu loss $= I_1^2 R_{01}$ or $I_2^2 R_{02} = W_{cu}$

Iron loss $=$ Hysteresis loss $+$ Eddy current loss $= W_h + W_e = W_i$

Considering primary side,

Primary input $= V_1 I_1 \cos \Phi_1$

$$\eta = \frac{V_1 I_1 \cos \Phi_1 - \text{losses}}{V_1 I_1 \cos \Phi_1} = \frac{V_1 I_1 \cos \Phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos \Phi_1} = 1 - \frac{I_1 R_{01}}{V_1 \cos \Phi_1} - \frac{W_i}{V_1 I_1 \cos \Phi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos \Phi_1} + \frac{W_i}{V_1 I_1^2 \cos \Phi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence, the above equation becomes

$$\frac{R_{01}}{V_1 \cos \Phi_1} = \frac{W_i}{V_1 I_1^2 \cos \Phi_1} \quad \text{or} \quad W_i = I_1^2 R_{01} \quad \text{or} \quad W_i = I_2^2 R_{02}$$

Or Cu loss $=$ Iron loss

The output current corresponding to maximum efficiency is $I_2 = \sqrt{(W_i/R_{02})}$

Load kVA corresponding to maximum efficiency, $=$ Full load $\times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L.Cu loss}}\right)}$

The efficiency at any load is given by $\eta = \frac{x \times \text{full-load kVA} \times \text{p.f.}}{(x \times \text{full-load kVA} \times \text{p.f.}) + W_{cu} x^2 + W_i} \times 100$

Where:

x = ratio of actual to full-load in kVA

W_i = iron loss in kW

W_{cu} = Cu loss in kW

Why Transformer Rating in *kVA*?

As seen, Cu loss of a transformer depends on current and iron loss on voltage. Hence, total transformer loss depends on volt-ampere (VA) and not on phase angle between voltage and current *i.e.* it is independent of load power factor. That is why rating of transformers is in kVA and not in kW.

Example 10: A 600kVA, 1-phase transformer has an efficiency of 92% both at full-load and half-load at unity power factor. Determine its efficiency at 60% of full-load at 0.8 power factor lag.

Solution:

$$\eta = \frac{x \times \text{full-load kVA} \times \text{p.f.}}{(x \times \text{full-load kVA} \times \text{p.f.}) + W_{cu} + W_i} \times 100 = \frac{x \times \text{kVA} \times \cos\Phi}{(x \times \text{kVA}) \times \cos\Phi + W_i + x^2 W_{cu}} \times 100$$

Where:

x = represents percentage of full-load in kVA

W_i = iron loss in kW ; W_{cu} = Cu loss in kW

At F.L. u.p.f.: Here $x = 1$

$$\therefore \quad 92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_1 + W_{cu}} \times 100, \; W_i + W_{cu} \cong 52.174\text{kW} \quad \ldots(i)$$

At half F.L. UPF: Here $x = \frac{1}{2}$

$$92 = \frac{(1/2) \times 600 \times 1}{(1/2) \times 600 \times 1 + W_i + (1/2)^2 W_{cu}} \times 100 \; ; \; W_i + 0.25 W_{cu} \cong 26.087\text{kW}$$

$\ldots(ii)$

From (i) and (ii), we get, $W_i = 17.39\text{kW}$; $W_{cu} = 34.78\text{kW}$

60% F.L. 0.8 p.f. (lag) Here, $x = 0.6$

$$\eta = \frac{0.6 \times 600 \times 0.8}{(0.6 \times 600 \times 0.8) + 17.39 + (0.6)^2 \times 34.78} \times 100 = 90.59\%$$

Example 11: A 11,000/230 V, 150kVA, 1-phase, 50Hz transformer has core loss of 1.4kW and F.L. Cu loss of 1.6kW. Determine:

(i) The kVA load for max. efficiency and value of max. efficiency at unity p.f.

(ii) The efficiency at half F.L. 0.8 p.f. leading

Solution:

(i) Load kVA corresponding to maximum efficiency is

$$= \text{F.L kVA} \times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L.Cu loss}}\right)} = 150 \times \sqrt{\frac{1.4}{1.6}} \cong 140\text{kVA}$$

Since Cu loss equals iron loss at maximum efficiency,

Total loss $= 1.4 + 1.4 = 2.8\text{kW}$, Output $= 140 \times 1 = 140\text{kW}$

$$\eta_{max} = \frac{140}{142.8} \cong 0.9804 \quad \text{or} \quad 98.04\%$$

(ii) Cu loss at half full-load $= 1.6 \times \left(\frac{1}{2}\right)^2 = 0.4\text{kW}$,

Total loss $= 1.4 + 0.4 = 1.8\text{kW}$

Half F.L. output at 0.8 p.f. $= \left(\frac{150}{2}\right) \times 0.8 = 60\text{kW}$

$$\therefore \quad \text{Efficiency} = \frac{60}{60 + 1.8} = 0.97 \quad \text{or} \quad 97.08\%$$