

# LECTURE 3 : AC-DC CONVERSION

## (RECTIFICATIONS)

### 3.1 Basic Rectifier Circuits

Several types of rectifier circuits are available: single-phase and three-phase half-wave and full-wave, controlled and uncontrolled, etc. For a given application, the type used is determined by the requirements of that application. In general the types of rectifiers are:

1. Uncontrolled Rectifier

Provide a fixed d.c. output voltage for a given a.c. supply where diodes are used only.

2. Controlled Rectifier

Provide an adjustable d.c. output voltage by controlling the phase at which the devices are turned on, where thyristors and diodes are used.

A. *Half-controlled*

allows electrical power flow from a.c. to d.c. (i.e. rectification only )

B. *Fully - controlled*

allow power flow in both directions (i.e. rectification and inversion )

### Application of Rectifier Circuits

- ☐ There are many applications for rectifiers. Some of them are:
  - »Variable speed dc drives,
  - »Battery chargers,
  - »DC power supplies for electric railways and
  - »Power supply for a specific application like electroplating
- ☐ Power rating of a single-phase rectifier tends to be lower than 10 kW.
- ☐ Three-phase bridge rectifiers are used for delivering higher power output, up to 500 kW at 500 V dc or even more.

In our study, we shall be interested mainly in single-phase half-wave and full—wave phase controlled rectifiers, but we shall also consider uncontrolled diode rectifiers to clarify the presentation.

**NOTE 1:** In the following analysis we shall neglect forward voltage drop, and reverse and forward leakage currents in diodes and thyristors, we shall also neglect transient switching effects related to the dynamic characteristics of the solid-state components

**NOTE 2:** We shall always assume that the pulse is of sufficient amplitude to switch on the appropriate thyristor.

### **3.2 Uncontrolled Rectifications**

In this type, the generated DC power is fixed with the converter used and the input AC power. They usually use diodes as their power switches. The following subsections deal with the basic operation of some examples of uncontrolled rectifiers single-phase half-wave rectifier loaded with resistive and resistive inductive loads.

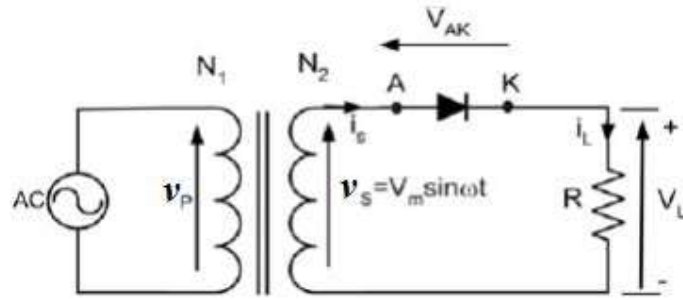
#### **3.2.1- Single-phase Half-Wave Uncontrolled Rectifier with resistive load**

Fig.3.1 shows the basic circuit for a single-phase, half-wave uncontrolled rectifier supplying a resistive load. The circuit is supplied by a single phase transformer whose secondary represents the rectifier's circuit AC source ( $v_s$ ) that is represented by a sinusoidal wave given by

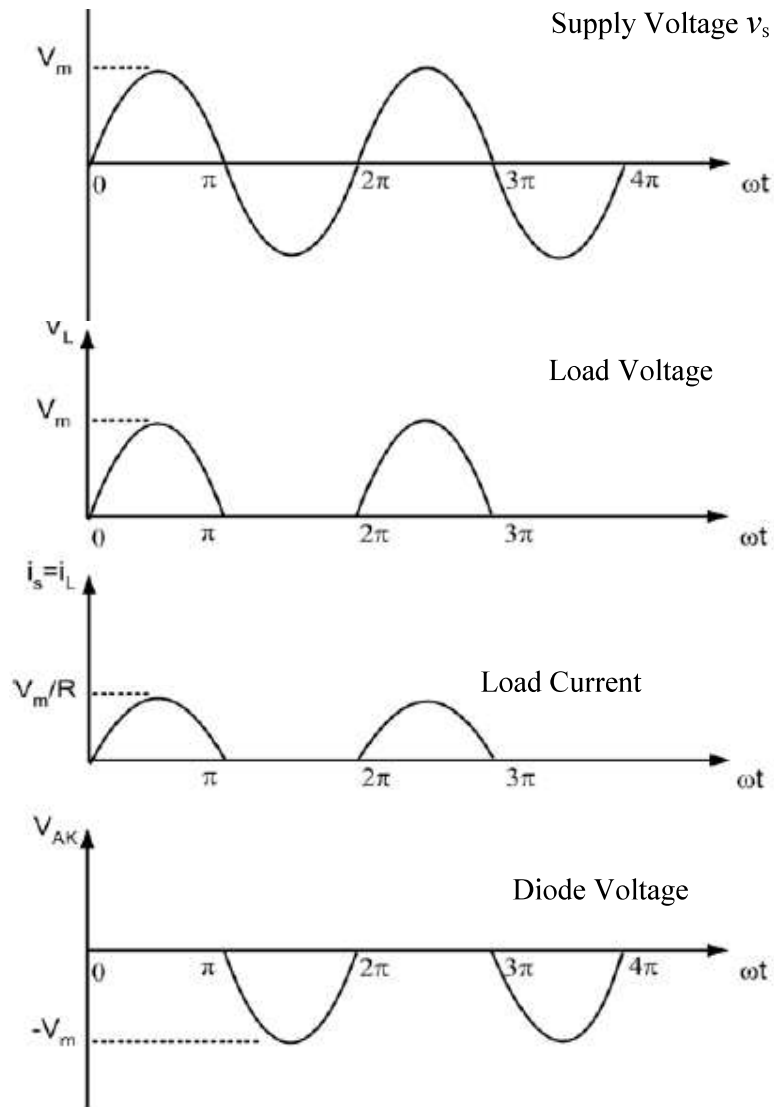
$$v_s = V_m \sin (\omega t) \quad (3.1)$$

where  $v_s$  is the supply voltage,  $V_m$  is the peak value of the supply voltage,  $\omega$  is the angular frequency, and  $t$  and is the time.

For this configuration, the diode will conducts (becomes forward biased) whenever the supply voltage ( $v_s$ ) is positive.



(a)



(b)

Fig.3. 1 Single-phase half-wave rectifier: a) circuit and b) waveforms

The average value of the load voltage  $V_{dc}$  can be calculated as follows:

$$V_{dc} = \frac{1}{2\pi} \int_0^\pi v_s(\omega t) d\omega t \quad (3.2)$$

$$= \frac{1}{2\pi} \int_0^\pi V_m \sin \omega t d\omega t = \frac{V_m}{2\pi} (-\cos \pi + \cos 0^\circ) = \frac{V_m}{\pi} \quad (3.3)$$

Since the load is resistive, therefore the load voltage and current are in phase and they are related by  $i = v / R$ . Consequently, the average value of the load current  $I_{dc}$  is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R} \quad (3.4)$$

The output DC power is given by:

$$P_{dc} = V_{dc} I_{dc} = \frac{V_{dc}^2}{R} \quad (3.5)$$

The rms value of the load voltage  $V_{orms}$  can be calculated as follows:

$$V_{orms} = \sqrt{\frac{1}{2\pi} \int_0^\pi v_s^2(\omega t) d\omega t} \quad (3.6)$$

$$= \sqrt{\frac{1}{2\pi} \int_0^\pi (V_m \sin \omega t)^2 d\omega t}$$

$$= \sqrt{\frac{(V_m)^2}{2\pi} \int_0^\pi \frac{1}{2} (1 - \cos 2\omega t) d\omega t} = \frac{V_m}{2} \quad (3.7)$$

$$\text{Since: } (\sin \omega t)^2 = \frac{1}{2} (1 - \cos 2\omega t)$$

Therefore the rms value of the load current  $I_{orms}$  is :

$$I_{orms} = \frac{V_{rms}}{R} = \frac{V_m}{2R} = \frac{I_m}{2} \quad (3.8)$$

The rms value  $I_{orms} = I_m/2$  for half – wave operation compared with the corresponding value for pure sinusoidal operation which is  $I_{orms} = I_m/\sqrt{2}$ .

The output a.c. power is given by:

$$P_{ac} = V_{orms} I_{orms} = \frac{V_{orms}^2}{R} = \frac{(0.5V_m)^2}{R} \quad (3.9)$$

### Performance parameters

The performance of a rectifier can be evaluated in terms of the following parameters:

The output d.c. power is given by

$$P_{dc} = V_{dc} I_{dc}$$

The output a.c. power is given by

$$P_{ac} = V_{orms} I_{orms}$$

3. The efficiency of the converter is given by

$$\eta = \frac{P_{dc}}{P_{ac}} \quad (3.10)$$

### 4. Output A.C. Component

The output voltage can be considered to have two components: including (i) d.c. value and (ii) the a.c. components or ripple. The rms value of the a.c. component of the output voltage is

$$V_{ac} = \sqrt{V_{orms}^2 - V_{dc}^2} \quad (3.11)$$

### 5. Form Factor, $FF$

It is a measure of the shape of the output voltage.

$$FF = \frac{V_{orms}}{V_{ac}} \quad (3.12)$$

### 6. Ripple factor, $RF$

It is a measure of the ripple content or the degree of distortion in a rectified voltage waveform which can be calculated as

$$\begin{aligned} \text{Ripple factor} &= \frac{\text{rms value of the ac components}}{\text{average value}} \\ &= \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} \end{aligned} \quad (3.13)$$

7. Transformer utilization factor,  $TUF$

$$TUF = \frac{P_{dc}}{V_s I_s} \quad (3.14)$$

where

$V_s$  : rms voltage of the transformer secondary

$I_s$  : rms current of the transformer secondary

### Example 3.1

An ideal single-phase source, 240 V, 50 Hz, supplies power to a load resistor  $R = 100 \Omega$  via a single ideal diode.

- (a) Calculate the average and rms values of the current and the power dissipation.
- (b) Calculate the circuit power factor and the ripple factor.
- (c) What must be the rating of the diode?

Solution:

$$I_{dc} = \frac{V_m}{\pi R} = \frac{240\sqrt{2}}{\pi \times 100} = 1.08A$$

$$I_{orms} = \frac{V_m}{2R} = \frac{240\sqrt{2}}{2 \times 100} = 1.7A$$

The average power dissipation in the load resistor  $R$  is given by the average a.c. power:

$$P_{ac} = I_{orms}^2 R = \left(\frac{V_m}{2R}\right)^2 = \frac{V_m^2}{4R^2} = (1.7)^2 \times 100 = 289 \text{ W}$$

$$P_{in} = V_s \times I_{orms} = 240 \times 1.7 = 408 \text{ W}$$

$$PF = \frac{P_{ac}}{P_{in}} = \frac{289}{408} = 0.708$$

$$\text{Ripple factor} = RF = \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} = \frac{\sqrt{I_{orms}^2 - I_{dc}^2}}{I_{dc}}$$
$$RF = \sqrt{\left(\frac{I_{orms}}{I_{dc}}\right)^2 - 1} = \sqrt{\left(\frac{1.7}{1.08}\right)^2 - 1} = 1.21$$

The value 1.25 for half – wave rectification is large, since the ideal value of the ripple factor should be zero for the output d.c. voltage. The diode must be rated in terms of a peak reverse voltage and a mean forward current.

Diode PRV  $= V_m = \sqrt{2} \times 240 = 339.4V$  , Choose 400V diode  
 Either the rms or the mean (average) current could be use as the basis of current rating: Since  $I_{orms} = 1.7A$  choose 2A diode rating.

### 3.2.2 . Single –phase half - wave uncontrolled rectifier with ( R-L) Load:

If the load consists of a series resistor and inductor, the current will flow through the negative cycle as well; Fig.3.2 shows the circuit diagram and Fig.3.3 shows the load voltage and current waveform for this case.

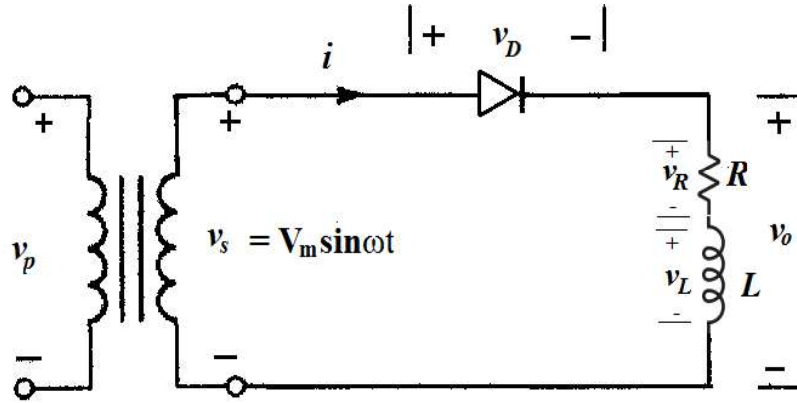


Fig .3.2 Single – phase half – wave rectifier with  $R-L$  load.

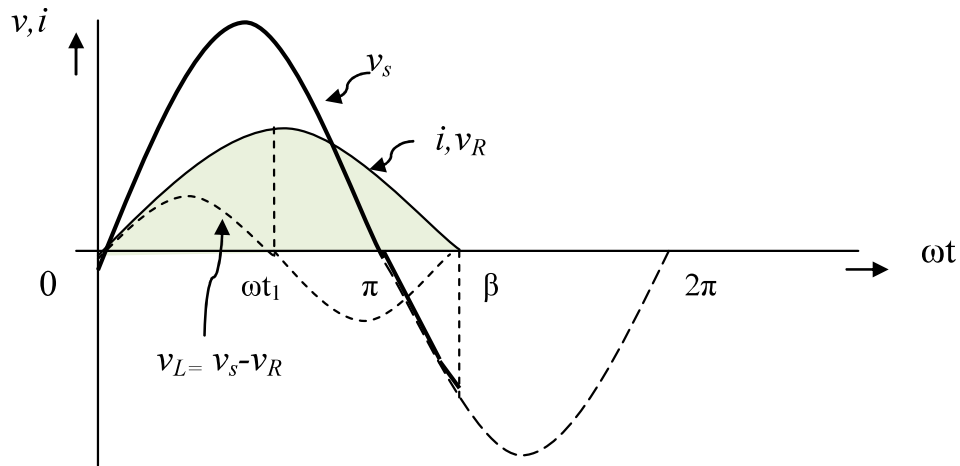


Fig.3.3 Waveforms for Fig.3.2.

During conduction period, we have by KVL;

$$v_s = v_o = v_R + v_L \quad (3.15)$$

$$V_m \sin \omega t = i R + L \frac{di}{dt} \quad (3.16)$$

Each supply period (cycle) in Fig. 3.3 can be divided into 4-distinct regions:-

- From 0-  $\omega t_1$ : the current rises from zero to peak, which lags the voltage peak due to circuit inductance;  $v_L$  is positive and the inductance store energy.
- From  $\omega t_1$  -  $\pi$ : the current decays, and hence  $v_L$  is negative. Both source and inductance supply energy to  $R$ .
- From  $\pi$  -  $\beta$ : the current continues to decay until it reaches zero,  $v_L$  remains negative, and hence energy is supplied by inductance to both source and resistor.
- From  $\beta$  -  $2\pi$ : at  $\beta$  current reaches zero and the diode cut- out. Current remains zero until the beginning of the next positive half cycle.

The average output voltage is

$$V_{dc} = \frac{1}{2\pi} \int_0^{\beta} v_s(\omega t) d(\omega t) = \frac{1}{2\pi} \int_0^{\beta} V_m \sin \omega t d(\omega t)$$

$$V_{dc} = \frac{V_m}{2\pi} \left[ -\cos \omega t \right]_0^{\beta} = \frac{V_m}{2\pi} [-\cos \beta + \cos 0] = \frac{V_m}{2\pi} [1 - \cos \beta] \quad (3.17)$$

The average output current is

$$I_{dc} = \frac{V_m}{2\pi R} [1 - \cos \beta] \quad (3.18)$$

### Equation of the current:

The equation for the current through R-L load can be found from the solution of the differential equation (3.16) which can be re-written as:

$$L \frac{di}{dt} + i R = V_m \sin \omega t \quad (3.19)$$

This is a first order differential equation. The solution of this equation has two parts:

1- Steady state solution:  $i_{ss}(\omega t) = \frac{V_m}{Z} \sin(\omega t - \theta)$

2- Transient solution :  $i_{tr}(\omega t) = A e^{-\frac{t}{\tau}}$

where

$$Z = \sqrt{R^2 + \omega^2 L^2};$$

$$\tan \theta = \frac{\omega L}{R}$$

$$\tau = \frac{L}{R} = \text{time constant}$$

$$A = \text{constant}$$

The complete solution is :  $i(\omega t) = i_{ss}(\omega t) + i_{tr}(\omega t)$

$$i(\omega t) = \frac{V_m}{Z} \sin(\omega t - \theta) + A e^{-\frac{t}{\tau}} \quad (3.20)$$

The constant A can be found from initial conditions: when  $t = 0$ ,  $i(\omega t) = 0$ ,

$$0 = \frac{V_m}{Z} \sin(-\theta) + A e^{-0}$$

$$\therefore A = \frac{V_m}{Z} \sin(\theta) \quad (3.21)$$

Substitute eq. (3.21) into eq. (3.20) yields

$$i(\omega t) = \frac{V_m}{Z} \sin(\omega t - \theta) + \frac{V_m}{Z} \sin(\theta) e^{-\frac{t}{\tau}}$$

The final current equation is

$$i(\omega t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} [\sin(\omega t - \theta) + \sin\theta e^{-\cot\theta \cdot \omega t}] \quad (3.22)$$

The extinction angle can be determined for a given load impedance angle  $\theta$  from the above current equation (3.22).

**To find  $\beta$  ,**

From the final condition of the current, when  $t = \beta$  ,  $i = 0$  , hence from above equation :

$$0 = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} [\sin(\beta - \theta) + \sin\theta e^{-\cot\theta \cdot \beta}]$$

$$0 = [\sin(\beta - \theta) + \sin\theta e^{-\cot\theta \cdot \beta}]$$

$$\sin(\beta - \theta) = -\sin\theta e^{-\cot\theta \cdot \beta} \quad (3.23)$$

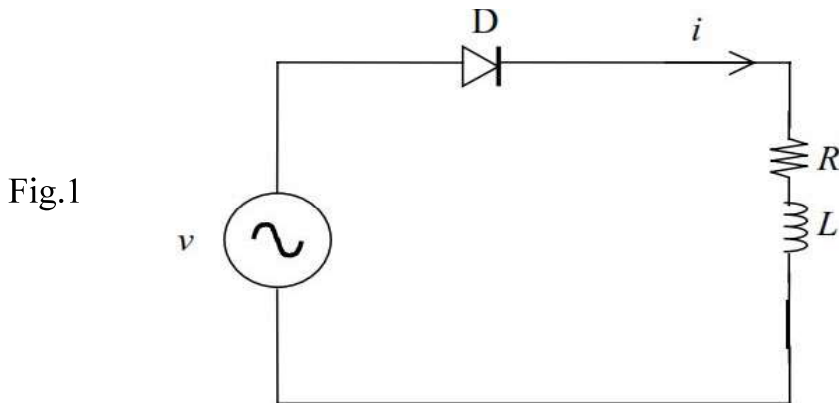
This is a transcendental equation which cannot be solved analytically. It can only be solved numerically by iteration technique as follows:

The initial guess of  $\beta$  is :  $\beta_o = 180^\circ + \theta + \Delta$  , where  $\Delta$  is few degrees.

### Example 3.2

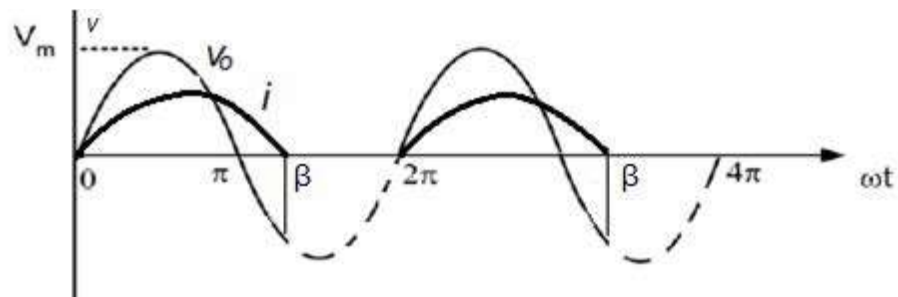
For the half-wave uncontrolled rectifier circuit supplying a series resistive-inductive load shown in Fig.1. The supply voltage is  $v = 300 \sin \omega t$  and the supply frequency is 50 Hz and the load parameter values are:  $R = 10 \Omega$ , and  $X_L = 20 \Omega$ .

- Sketch the load voltage and current waveforms.
- Calculate the current extinction angle  $\beta$ .
- Calculate the average DC load voltage  $V_{dc}$ .
- Calculate the angle of maximum current.



Solution:

- The output voltage and current waveforms are:



- It is given that:

$$i(\omega t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} [\sin(\omega t - \theta) + \sin \theta e^{-\cot \theta \cdot \omega t}]$$

$$\text{where } Z = \sqrt{R^2 + (\omega L)^2} \quad \tan \theta = \omega L / R$$

To find  $\beta$ ,

From the final condition of the current, when  $t = \beta$ ,  $i = 0$ , hence from above equation :

$$0 = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} [\sin(\beta - \theta) + \sin\theta e^{-\cot\theta \cdot \beta}]$$

$$0 = [\sin(\beta - \theta) + \sin\theta e^{-\cot\theta \cdot \beta}]$$

$$0 = \sin(\beta - \theta) + \sin\theta e^{-\cot\theta \cdot \beta}$$

This is a transcendental equation which can be solved numerically by iteration technique as follows:

The initial guess of  $\beta$  is :  $\beta_o = 180^\circ + \theta + \Delta$ , where  $\Delta$  is few degrees.

$$\tan \theta = \omega L / R = X_L / R = 20 / 10 = 2$$

hence  $\theta = 63.4^\circ$ , and therefore :  $\beta_o = 180^\circ + \theta + \Delta = 180^\circ + 63.4^\circ + \Delta = 243.4^\circ + \Delta$

as a starting value for iteration :

$\beta_o = 246^\circ$  ( By choosing  $\Delta = 2.6^\circ$  ) :

| <u>Estimated <math>\beta</math></u> |               | $\sin(\beta - \theta) + \sin\theta e^{-\cot\theta \cdot \beta}$ |
|-------------------------------------|---------------|-----------------------------------------------------------------|
| $\beta$ (degrees)                   | $\beta$ (rad) |                                                                 |
| 246                                 | 4.294         | - 0.033                                                         |
| 248 <sup>o</sup>                    | 4.33          | 0.023                                                           |
| 250 <sup>o</sup>                    | 4.363         | - 0.014                                                         |
| 249 <sup>o</sup>                    | 4.316         | 0.0044                                                          |
| 249.25 <sup>o</sup>                 | 4.350         | - 0.003                                                         |

By intuition :  $\beta = 249.25^\circ$ . By intuition

(c) The average DC output voltage is :

$$V_{dc} = \frac{1}{2\pi} \int_0^\beta v_s(\omega t) d(\omega t) = \frac{1}{2\pi} \int_0^\beta V_m \sin\omega t d(\omega t)$$

$$V_{dc} = \frac{V_m}{2\pi} [-\cos\omega t]_0^\beta = \frac{V_m}{2\pi} [-\cos\beta + \cos 0] = \frac{V_m}{2\pi} [1 - \cos\beta]$$

Where  $V_m = 300$  V

Hence

$$V_{dc} = \frac{300}{2\pi} [1 - \cos 249.25] = 64.694 \text{ V}$$

(c) Angle of maximum current can be found as ,

The equation of the current (3.22) is

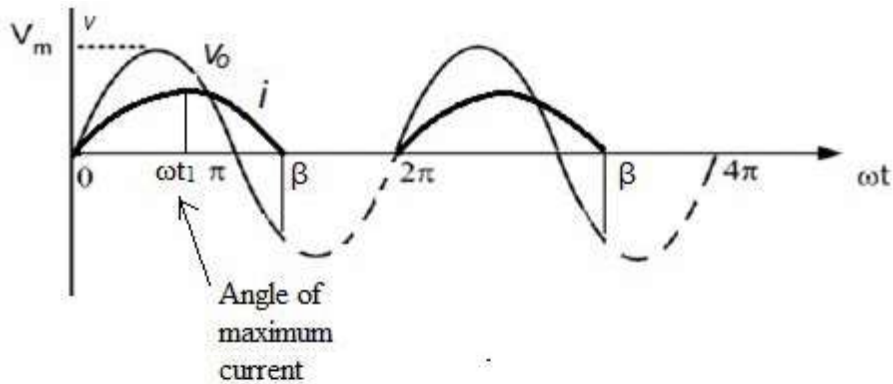
$$i(\omega t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} [\sin(\omega t - \theta) + \sin \theta e^{-\cot \theta \cdot \omega t}]$$

To find the maximum value of the current  $\frac{di(\omega t)}{dt} = 0$

$$\frac{di(\omega t)}{dt} = \frac{V_m}{\sqrt{10^2 + (20)^2}} [\omega \cos(\omega t - 63.4) - \omega \sin 63.4 e^{-\cot 63.4 \cdot \omega t}] = 0$$

$$\frac{di(\omega t)}{dt} = [\cos(\omega t - 63.4) - \sin 63.4 e^{-\cot 63.4 \cdot \omega t}] = 0$$

Maximum current occurs at  $\omega t = \omega t_1$  as shown in the following figure:



Hence,

$$\cos(\omega t_1 - 63.4) = \sin 63.4 e^{-\cot 63.4 \omega t_1}$$

Solve this equation numerically to find  $\omega t_1$ .