Refrigeration

6: Air Refrigeration Cycles

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Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

Bell-Coleman air refrigeration machine was developed by Bell-Coleman and Light Foot by reversing the Joule's air cycle. It was one of the earliest types of refrigerators used in ships carrying frozen meat. Fig.2.8 shows a schematic diagram of such a machine which consists of a compressor, a cooler, an expander and a refrigerator.

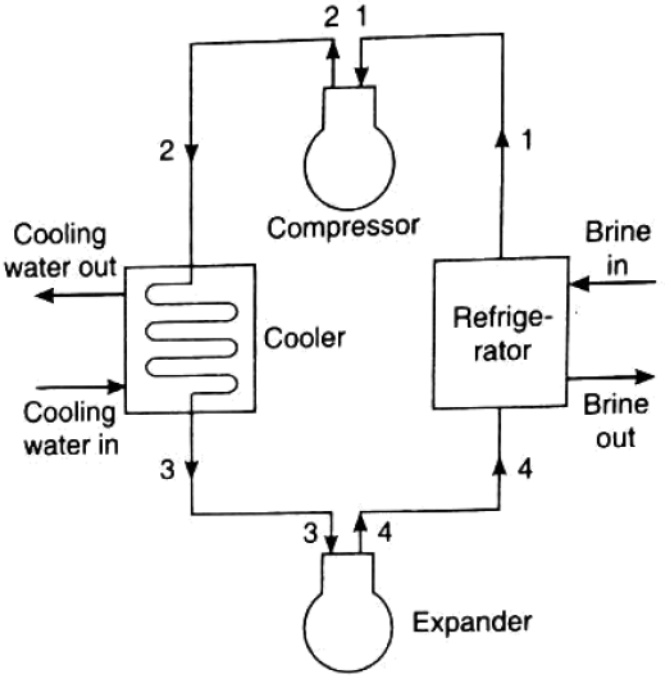
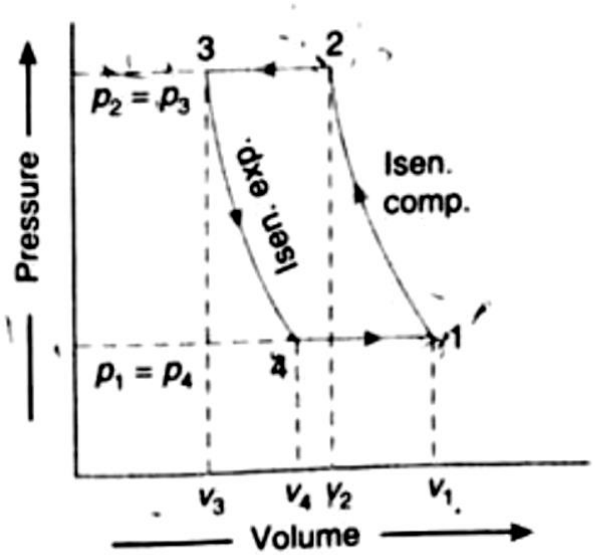


Fig. 2.8. Closed cycle or dense air Bell-Coleman Refrigerator.

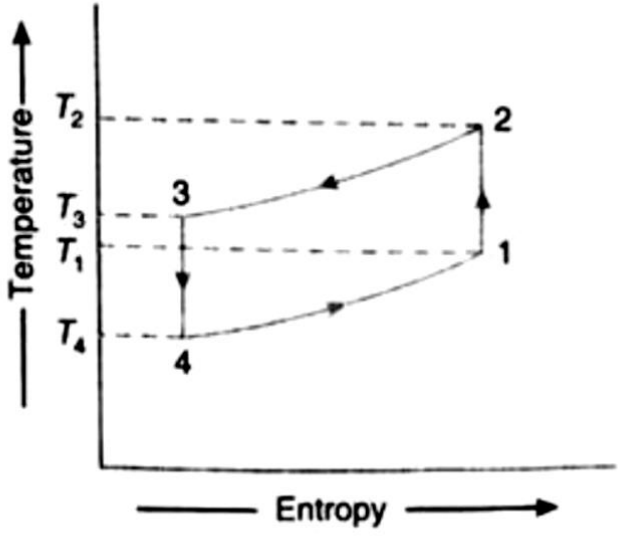
Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

The cycle is shown on **p-v** and **T-s** diagrams in Fig**.(a) and (b)**. At point 1, let p_1, v_1 and T_1 be the pressure, volume and temperature of air respectively. The four processes of the cycle are as follows:

Isentropic compression process. The cold air from the refrigerator is drawn into the compressor cylinder where it is compressed isentropically in the compressor as shown by the curve 1-2 on **p-v** and **T-s** diagrams. During the compression stroke, both the pressure and temperature increases and the specific volume of air at delivery from compressor reduces from v_1 to v_2. **We know that during isentropic compression process, no heat is absorbed or rejected by the air.**



(*a*) *p-v* diagram.

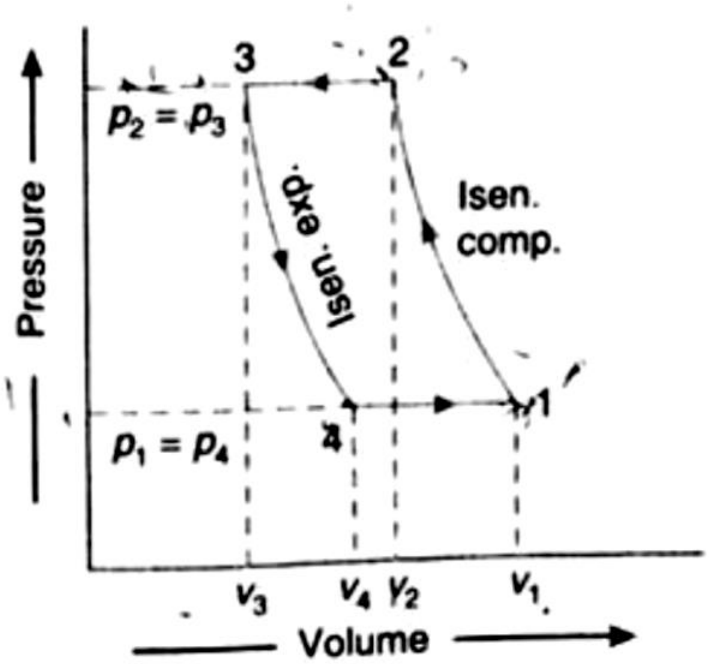


(*b*) *T-s* diagram.

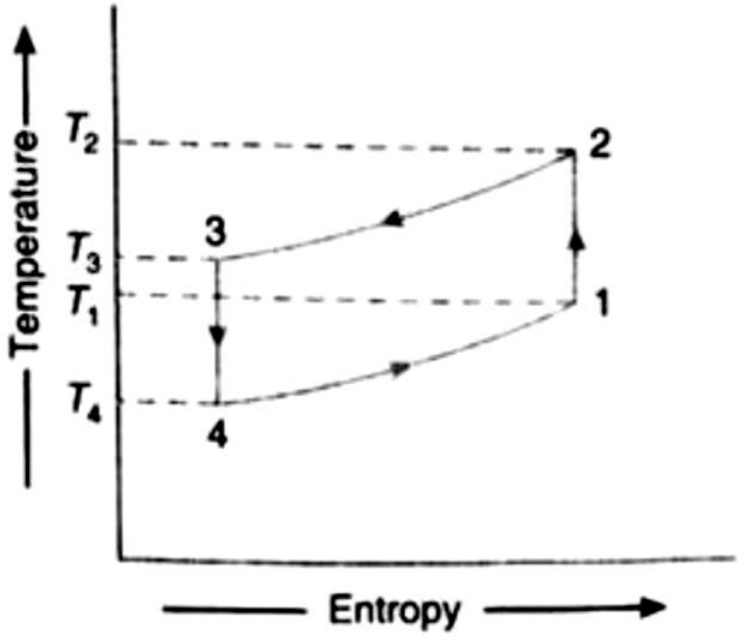
Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

Constant pressure cooling process. The warm air from the compressor is now passed from compressor to cooler where it is cooled at constant pressure p_3 (equal to p_2), reducing the temperature from T_2 to T_3 (the temperature of cooling water) as shown by the curve 2-3 on p-v and T-s diagrams. The specific volume also reduces from v_2 to v_3. We know that heat rejected by the air during constant pressure per kg of air,

$$\mathbf{q_r = q_{2\text{-}3} = c_p\,(T_2 - T_3)}$$



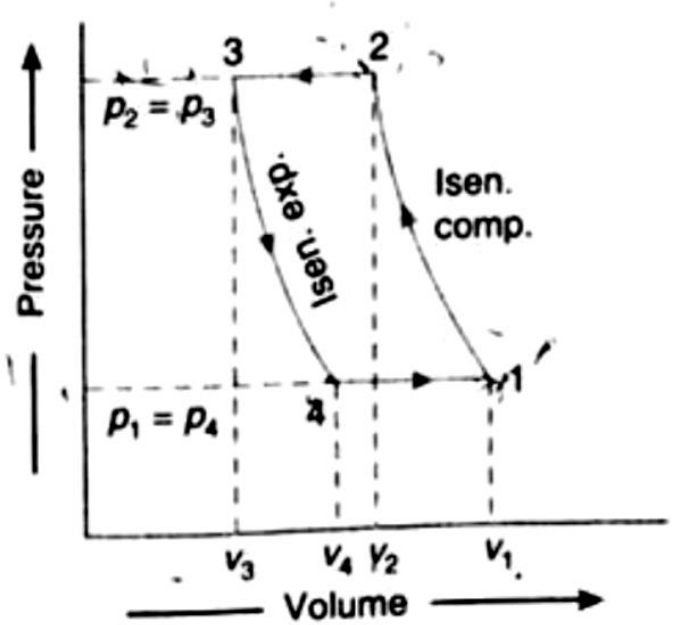
(*a*) *p-v* diagram.



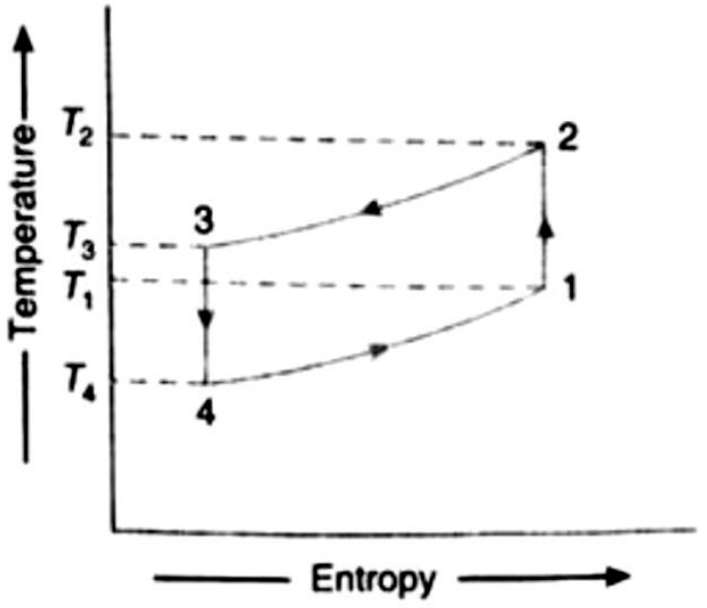
(*b*) *T-s* diagram.

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

Isentropic expansion process. The air from the cooler is now drawn into the expander cylinder where it is expanded isentropically from pressure P_3 to the refrigerator pressure P_4 which is equal to the atmospheric pressure. The temperature of air during expansion falls from T_3 to T_4 (i.e. the temperature much below the temperature of cooling water, T_3). The expansion process is shown by the curve 3-4 on the p-v and T-s diagrams. The specific volume of air at entry to the refrigerator increases from v_3 to v_4. We know that during isentropic expansion of air, no heat is absorbed or rejected by the air.



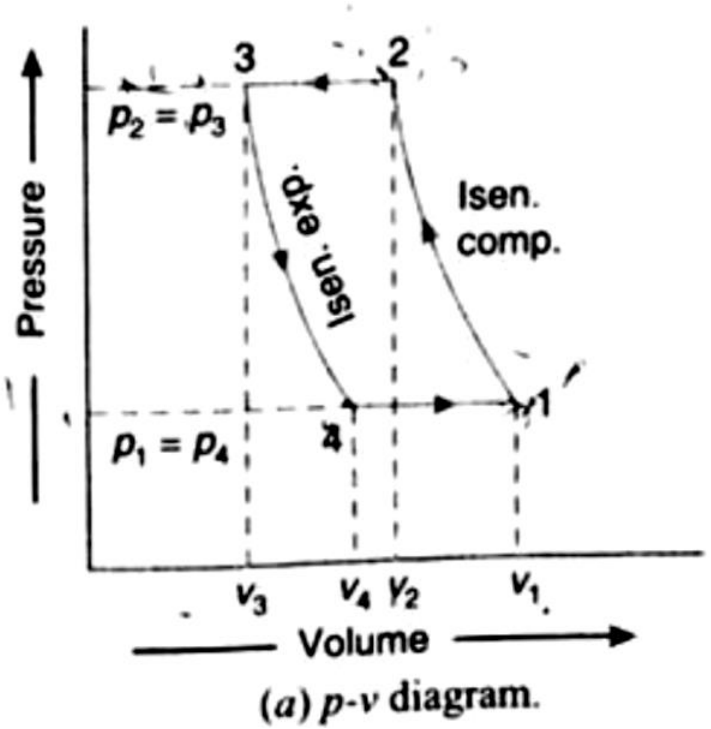
(*a*) *p-v* diagram.



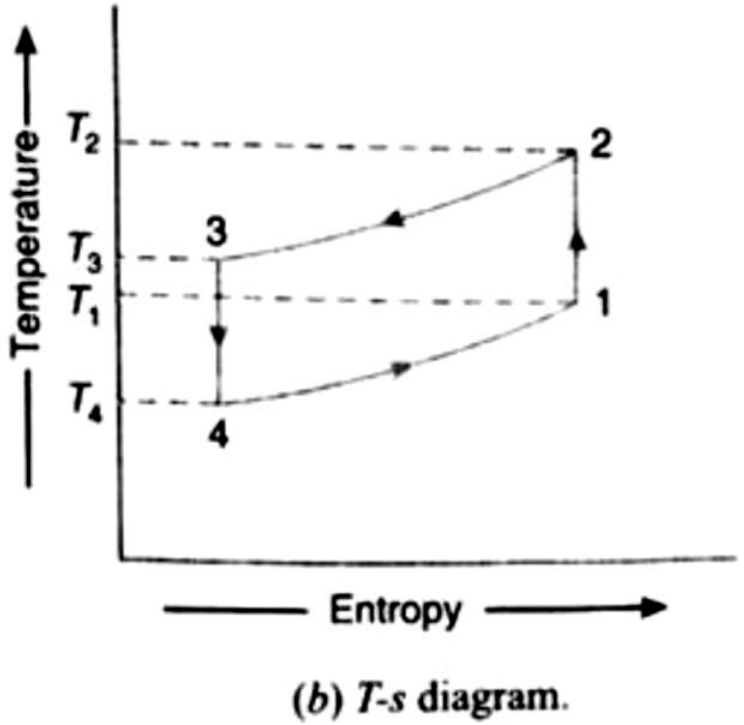
(*b*) *T-s* diagram.

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

Constant pressure expansion process. The cold air from the expander is now passed to the **refrigerator** where it is expanded at constant pressure p_4 (equal to p_1). The temperature of air increases from T_4 to T_1. This process is shown by the curve 4-1 on the p-v and T-s diagrams. Due to heat from the refrigerator, the specific volume of the air changes from v_4 to v_1. We know that the heat absorbed by the air (or heat extracted from the refrigerator or the refrigerating effect produced) during constant pressure expansion per kg of air is :



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

$$q_A = q_{4\text{-}1} = c_p (T_1 - T_4)$$

We know that work done during the cycle per kg of air

$$= \text{Heat rejected} - \text{Heat absorbed} = q_R - q_A$$

$$= c_p (T_2 - T_3) - c_p (T_1 - T_4)$$

$\therefore$ Coefficient of performance,

$$\text{C.O.P.} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{q_R - q_A} = \frac{c_p (T_1 - T_4)}{c_p (T_2 - T_3) - c_p (T_1 - T_4)}$$

$$= \frac{(T_1 - T_4)}{(T_2 - T_3) - (T_1 - T_4)}$$

$$= \frac{T_4 \left(\frac{T_1}{T_4} - 1 \right)}{T_3 \left(\frac{T_2}{T_3} - 1 \right) - T_4 \left(\frac{T_1}{T_4} - 1 \right)} \quad \ldots (i)$$

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

We know that for isentropic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots (ii)$$

Similarly, for isentropic expansion process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots (iii)$$

Since, $p_2 = p_3$ and $p_1 = p_4$, therefore from equations (*ii*) and (*iii*),

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_2}{T_3} = \frac{T_1}{T_4} \quad \ldots (iv)$$

Now substituting these values in equation (*i*), we get

$$\text{C.O.P.} = \frac{T_4}{T_3 - T_4} = \frac{1}{\frac{T_3}{T_4} - 1}$$

$$= \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}} - 1} \quad \ldots (v)$$

where r_p = Compression or Expansion ratio = $\frac{p_2}{p_1} = \frac{p_3}{p_4}$

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

Sometimes, the compression and expansion processes take place according to the law pv^n = Constant. In such a case, the C.O.P. is obtained from the fundamentals as discussed below :

We know that work done by the compressor during the process 1-2 per kg of air,

$$w = w_C - w_E = \frac{n}{n-1} \times R\left[(T_2 - T_1) - (T_3 - T_4)\right]$$

We also know that heat absorbed during constant pressure process 4-1,

$$= c_p (T_1 - T_4)$$

$$\therefore \quad \text{C.O.P.} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{w} = \frac{c_p (T_1 - T_4)}{\frac{n}{n-1} \times R\left[(T_2 - T_1) - (T_3 - T_4)\right]} \quad \ldots (vi)$$

We know that $R = c_p - c_v = c_v(\gamma - 1)$

Substituting the value of R in equation (*vi*),

$$\text{C.O.P.} = \frac{c_p (T_1 - T_4)}{\frac{n}{n-1} \times c_v (\gamma - 1)\left[(T_2 - T_1) - (T_3 - T_4)\right]}$$

$$= \frac{\gamma (T_1 - T_4)}{\frac{n}{n-1} \times (\gamma - 1)\left[(T_2 - T_1) - (T_3 - T_4)\right]} \quad \ldots \left[\because \frac{c_p}{c_v} = \gamma\right]$$

$$= \frac{T_1 - T_4}{\frac{n}{n-1} \times \frac{(\gamma - 1)}{\gamma}\left[(T_2 - T_3) - (T_1 - T_4)\right]} \quad \ldots (vii)$$

Notes : 1. In this case, the values of T_2 and T_4 are to be obtained from the following relations:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}}$$

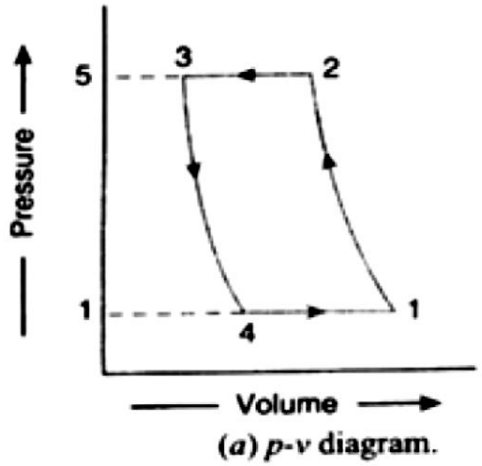
Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

Example: In a refrigeration plant working on Bell Coleman cycle, air is compressed to 5 bar from 1 bar. Its initial temperature is 10°C. After compression, the air is cooled up to 20°C in a cooler before expanding back to a pressure of 1 bar. Determine the theoretical C. O.P. of the plant and net refrigerating effect. Take c_p = 1.005 kJ /kg K and c_v = 0.718 kJ/kg K.

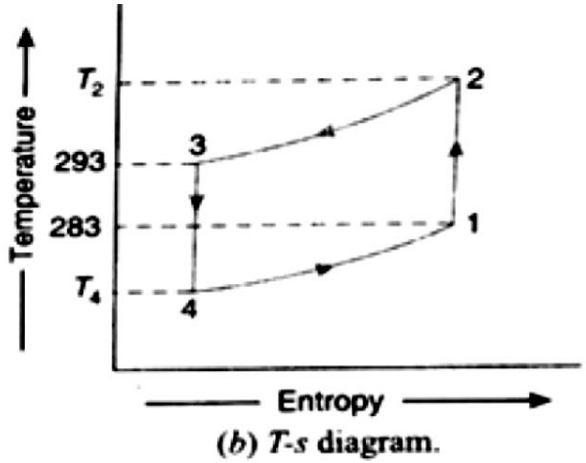
Solution. Given : $p_2 = p_3 = 5$ bar; $p_1 = p_4 = 1$ bar; $T_1 = 10°C = 10 + 273 = 283$ K ; $T_3 = 20°C = 20 + 273 = 293$ K; cp = 1.005 kJ/kg K; cv= 0.718 kJ/kg K

The p-v and T-s diagrams for a refrigeration plant working on Bell-Coleman cycle, is shown in Fig. (a) and (b) respectively.

Let T_2 and T_4 = Temperature of air at the end of compression and expansion respectively.

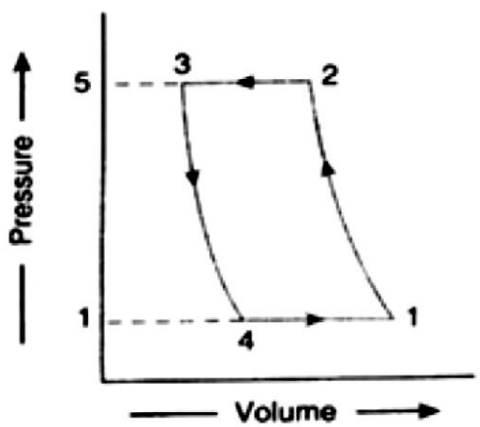


(a) p-v diagram.

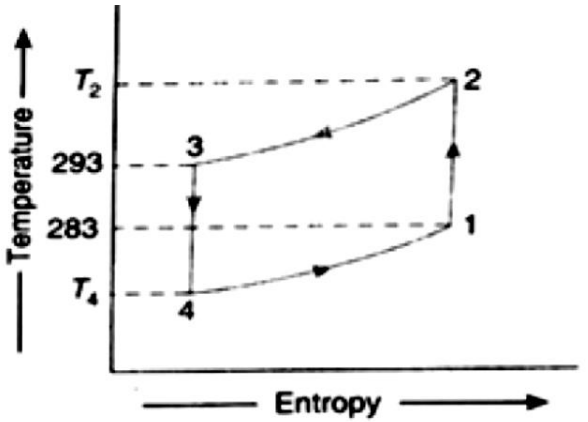


(b) T-s diagram.

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

We know that isentropic index for compression and expansion process,

$$\gamma = c_p / c_v = 1.005/0.718 = 1.4$$

For isentropic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.584$$

and for isentropic expansion process 3-4

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.584$$

$$\therefore \quad T_4 = T_3 / 1.584 = 293/1.584 = 185 \text{ K}$$

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

We know that *theoretical C.O.P. of the plant,

$$= \frac{T_4}{T_3 - T_4} = \frac{185}{293 - 185} = 1.713 \text{ Ans.}$$

We know that net refrigerating effect (*i.e.* heat absorbed during constant pressure process 4–1)

$$= c_p (T_1 - T_4) = 1.005 \ (283 - 185) = 98.5 \text{ kJ/kg Ans.}$$

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

Example: A refrigerator working on Bell-Coleman cycle operates between pressure limits of 1.05 bar and 8.5 bar. Air is drawn from the cold chamber at 10°C, compressed and then it is cooled to 30°C before entering the expansion cylinder. The expansion and compression follows the law $pv^{1.3}$= constant. Determine the theoretical C.O.P. of the system.

Solution. Given : $p_1 = p_4 = 1.05$ bar; p_2: = $p_3 = 8.5$ bar; $T_1 = 10°C = 10 + 273 = 283$ K ; $T_3 = 30°C = 30 + 273 = 303$ K ; $n = 1.3$

The p-v and T-s diagrams for a refrigerator working on the Bell-Coleman cycle is shown in Fig.
(a) and (b) respectively.

Let T_2 and T_4 = Temperature of air at the end of compression and expansion respectively. Since the compression and expansion follows the law $pv^{1.3} = C$, therefore

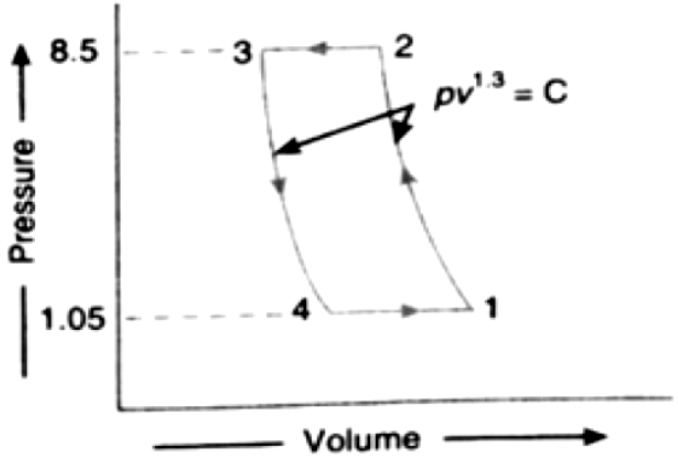
Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{8.5}{1.05}\right)^{\frac{1.3-1}{1.3}} = (8.1)^{0.231} = 1.62$$

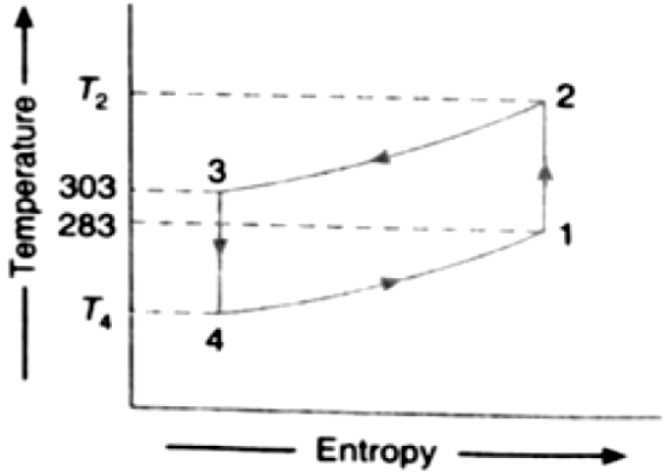
$$\therefore \quad T_2 = T_1 \times 1.62 = 283 \times 1.62 = 458.5 \text{ K}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{8.5}{1.05}\right)^{\frac{1.3-1}{1.3}} = 1.62$$

$$\therefore \quad T_4 = T_3/1.62 = 303/1.62 = 187 \text{ K}$$



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

We know that theoretical coefficient of performance,

$$\text{C.O.P.} = \frac{T_1 - T_4}{\frac{n}{n-1} \times \frac{(\gamma - 1)}{\gamma}\left[(T_2 - T_3) - (T_1 - T_4)\right]}$$

$$= \frac{(283 - 187)}{\frac{1.3}{1.3 - 1} \times \frac{(1.4 - 1)}{1.4}\left[(458.5 - 303) - (283 - 187)\right]} \quad \text{...(Taking } \gamma = 1.4\text{)}$$

$$= \frac{96}{1.24 \times 59.5} = 1.3 \text{ Ans.}$$

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

Example: The atmospheric air at pressure 1 bar and temperature -5°C is drawn in the cylinder of the compressor of a Bell-Coleman refrigerating machine. It is compressed isentropically to a pressure of 5 bar. In the cooler, the compressed air is cooled to 15°C, pressure remaining the same. It is then expanded to a pressure of 1 bar in an expansion cylinder, from where it is passed to the cold chamber.

Find : 1. the work done per kg of air, and 2. C. O.P. of the plant.

For air assume law for expansion, $pv^{1.2}$ =constant; law for compression, $pv^{1.4}$ =constant and specific heat of air at constant pressure = 1 kj/kg K.

Solution. Given : $p_1 = p_4 = 1$ bar ; T1 = - 5°C = - 5 + 273 = 268 K ; $p_2 = p_3 = 5$ bar; T3 = 15°C = 15 + 273 = 288 K; n = 1.2 ; $\gamma = 1.4$; cp = 1 kJ/kg K

The p-v and T-s diagrams for a refrigerating machine working on Bell-Coleman cycle is shown in Fig. (a) and (b) respectively.

Work done per kg of air.

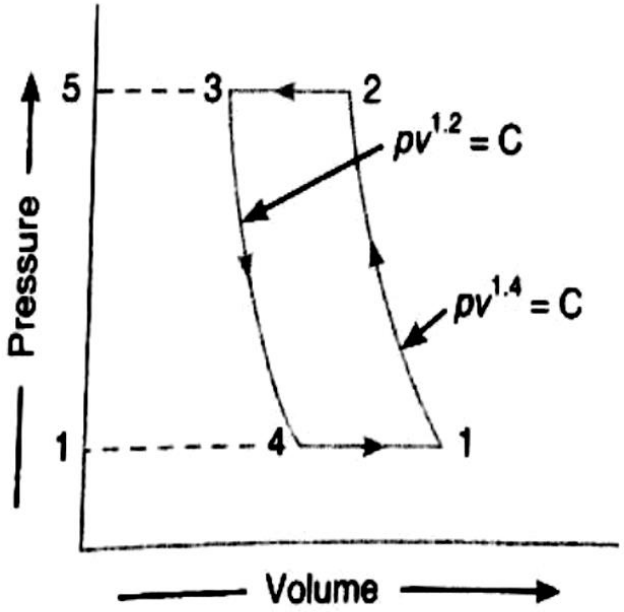
Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

The compression process 1-2 is isentropic and follows the law pv^n=constant.

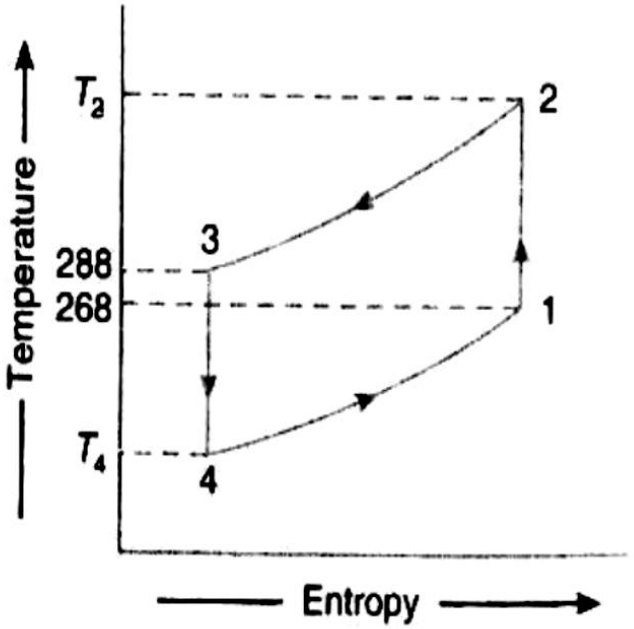
$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.585$$

or $$T_2 = T_1 \times 1.585 = 268 \times 1.585 = 424.8 \text{ K}$$

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

The expansion process 3-4 follows the law $pv^{1.2}$ = constant.

$$\therefore \quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.2-1}{1.2}} = (5)^{0.167} = 1.31$$

or $$T_4 = T_3/1.31 = 288/1.31 = 220 \text{ K}$$

We know that workdone by the compressor during the isentropic process 1-2 per kg of air,

$$w_C = w_{1\text{-}2} = \frac{\gamma}{\gamma - 1} \times R\,(T_2 - T_1)$$

$$= \frac{1.4}{1.4 - 1} \times 0.287(424.8 - 268) = 159 \text{ kJ/kg}$$

... (Taking R for air = 0.287 kJ/kg K)

and workdone by the expander during the process 3-4 per kg of air,

$$w_E = w_{3\text{-}4} = \frac{n}{n-1} \times R\,(T_3 - T_4)$$

$$= \frac{1.2}{1.2 - 1} \times 0.287(288 - 220) = 118.3 \text{ kJ/kg}$$

∴ Net work done per kg of air,

$$w = w_C - w_E = 159 - 118.3 = 40.7 \text{ kJ/kg Ans.}$$

2. *C.O.P. of the plant*

We know that heat absorbed during constant pressure process 4-1 per kg of air,

$$q_A = c_p(T_1 - T_4) = 1(268 - 220) = 48 \text{ kJ/kg}$$

$$\therefore \quad \text{C.O.P. of the plant} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{w} = \frac{48}{40.7} = 1.18 \text{ Ans.}$$

Next: Simple Air-cooled system

Thank you